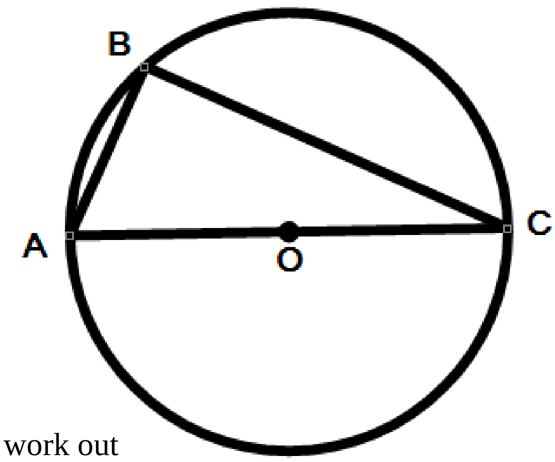
Circle Theorems GCSE Higher KS4 with Answers/Solutions

NOTE: You must give reasons for any answers provided.

All diagrams are NOT DRAWN TO SCALE.

1. (a) A, B and C are points on the circumference of a circle, centre, O.
AC is the diameter of the circle.

Write down the size of angle ABC.



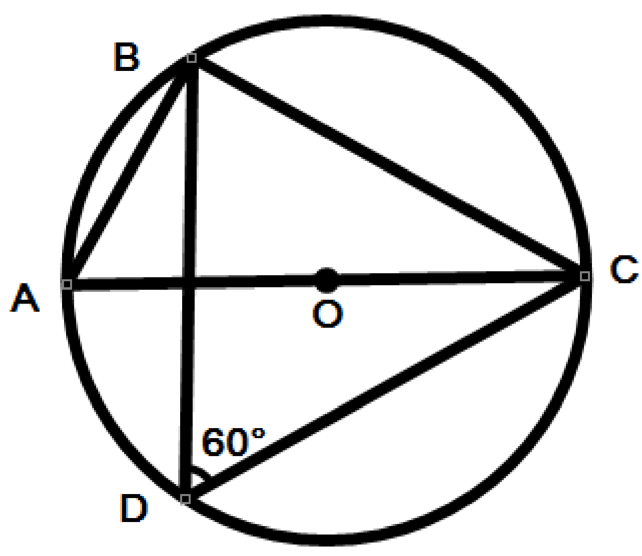
* (b) Given that AB = 6cm and BC = 8cm, work out

(i) the diameter of the circle, (ii) the area of the triangle

(iii) the area **and** circumference of the circle, **leaving your answer in terms of π.**

(c) D is a point on the circumference of the circle above such that angle BDC = 60˚.

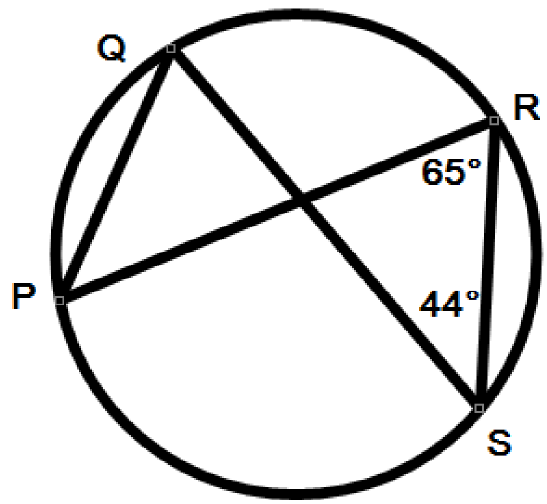
(i) Write down the size of angle CAB. (ii) Work out the size of angle ACB.



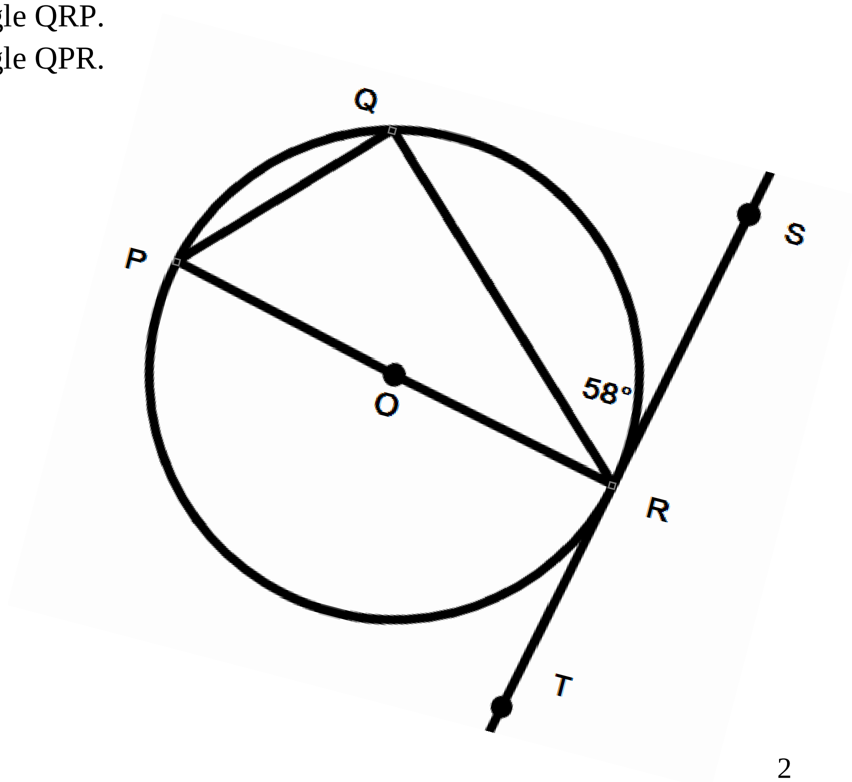
2. P, Q, R and S are points on the circumference of a circle.

   Angle PRS = 65˚ and angle QSR = 44˚.

   Find the size of angle (i) PQS (ii) QPR



3. P, Q and R are points on the circumference of a circle, centre, O.
   PR is the diameter of the circle and ST is a tangent to the circle at the point R.
   Angle QRS = 58˚.
   (a) Work out the size of angle QRP.
   (b) Work out the size of angle QPR.



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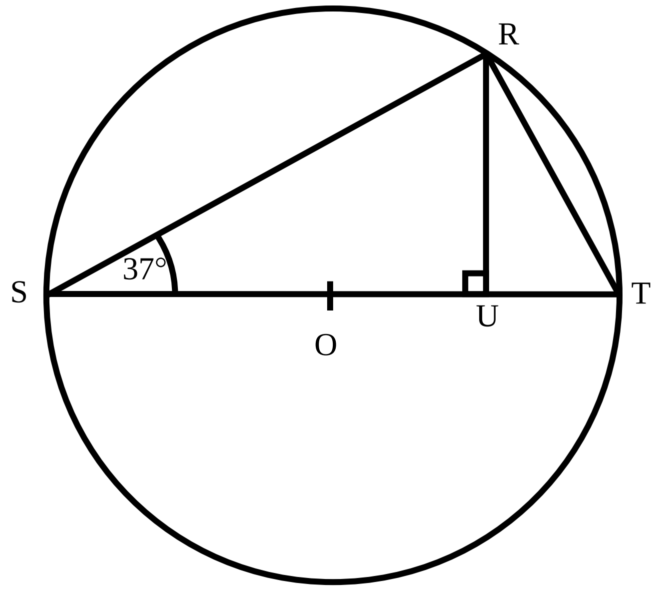


R, S and T are points on the circumference of a circle, centre *O*.

ST is a diameter and Angle RST = 37°.

U is the point on ST such that angle RUS is a right angle.

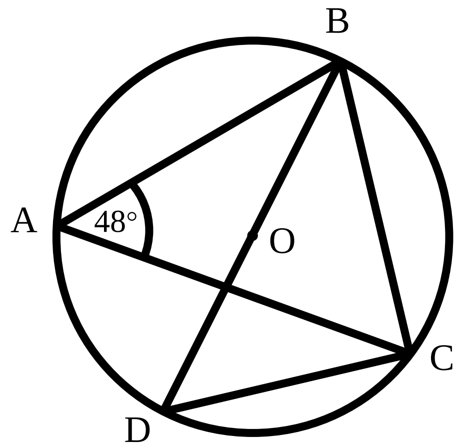
(*a*) Work out the size of angle URT.

(*b*) Work out the size of angle ROT.

(*c*) Work out the size of angle ORU.

(d) Find the size of angle ORT.

5.



A, B, C and D are points on the circumference of a circle, centre *O*.

BD is a diameter of the circle.   Angle CAB = 48°.

(a) Write down the size of angle BCD.

(b) Find the size of angle BDC.

(c) Find the size of angle BOC.

(d) Find the size of angle CAD.

(e) Find the size of angle COD.

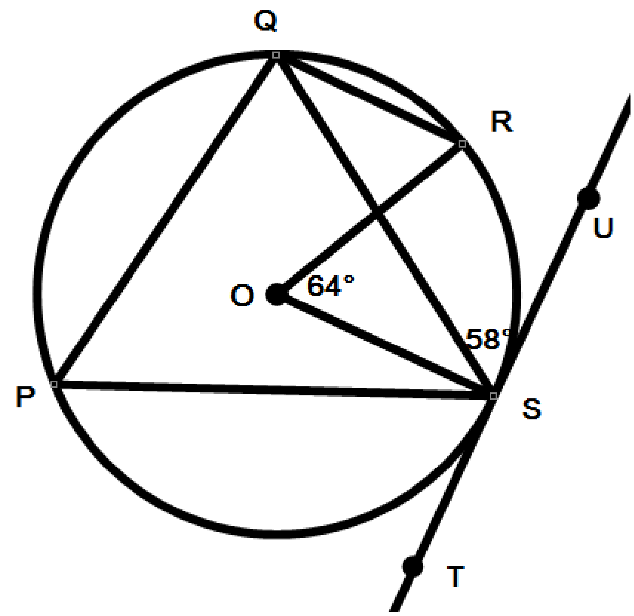
(f) Find the size of angle OCB.

6. P, Q, R and S are points on the circumference of a circle, centre, O.

TU is a tangent to the circle at the point S.

Angle ROS = 64˚ and angle QSU = 58˚.

(i) Find the size of angle:



(a) OSQ

(b) SQR

(c) QPS

(d) QRS

(ii) Why are the lines QR and OS parallel?

(iii) Find the size of angle (a) QRO (b) QSR

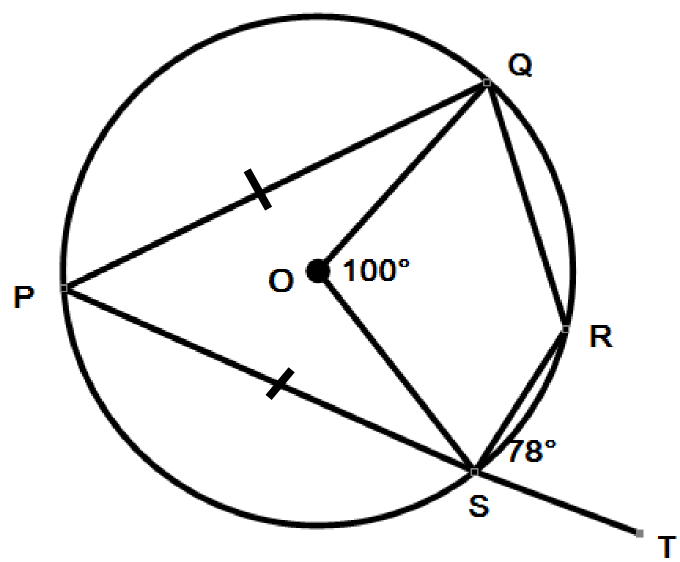
7. P, Q, R and S are points on the circumference of a circle, centre, O.

PST is a straight line.

PQ = PS

Angle SOQ = 100˚ and angle RST = 78˚

Work out the size of angle:



(a) QRS

(b) PQS

(c) OQS

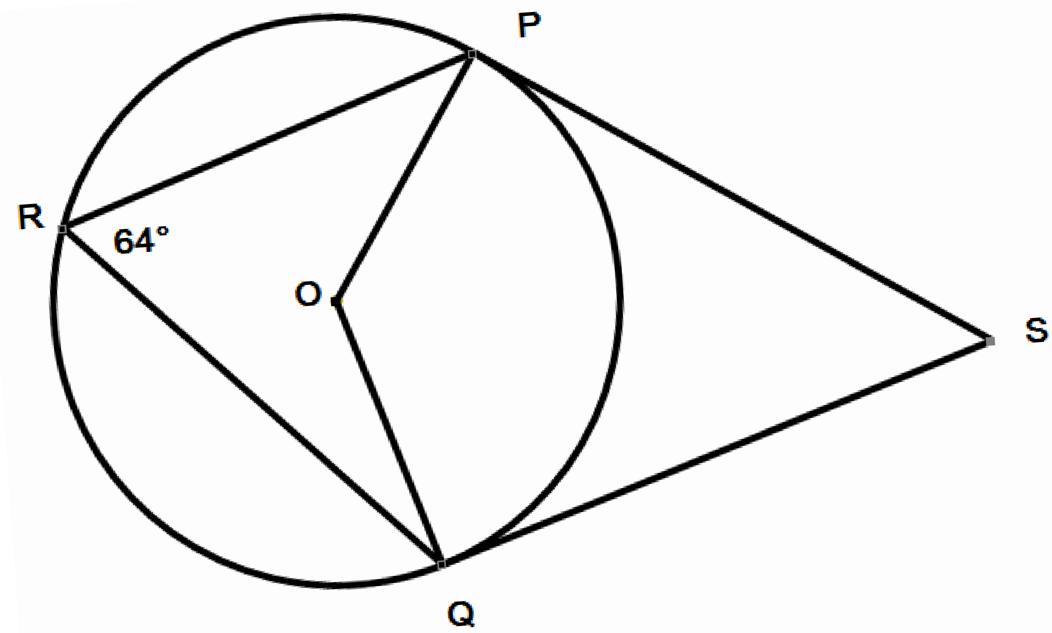
(d) PSO

(e) SQR

8. P, Q and R are points on the circumference of a circle, centre, O.

Angle PRQ = 64˚. SP and SQ are tangents to the circle at the points P and Q respectively.

Work out the size of angle (i) PSQ (ii) PQO (iii) POS (iv) QSO



9. P, Q and R are points on the circumference of a circle, centre, O.

Angle PSQ = 60˚. SP and SQ are tangents to the circle at the points P and Q respectively.

(a) Work out the size of angle:

(i) QPS

(ii) PQO

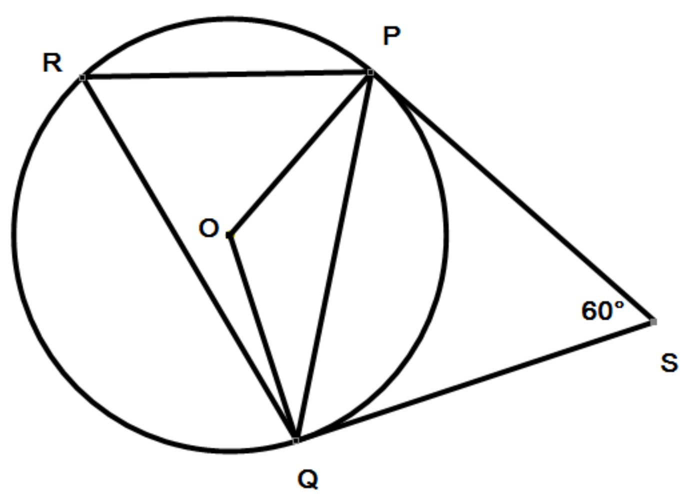
(iii) PRQ

(iv) POQ

(b) What type of triangle is PQS?

(c) Given that angle OQR = 10˚,

work out the size of angle OPR.



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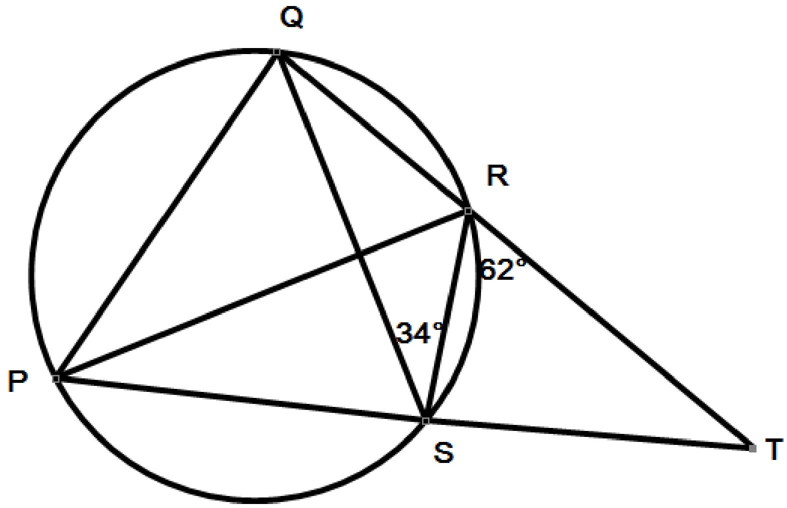


P, Q, R and S are points on the circumference of a circle.

PST and QRT are straight lines.

Angle QSR = 34˚ and angle SRT = 62˚.

(a) Find the size of the angle:

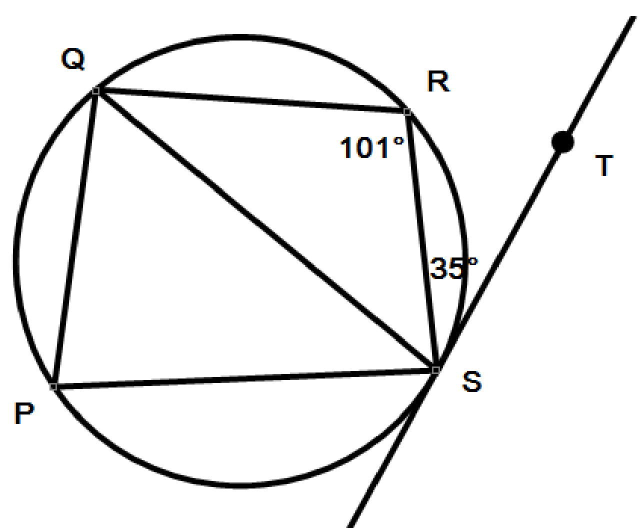
(i) SQR (ii) RPS

(b) Given that angle PRS = 62˚, show that PR is a diameter of the circle.

11. P, Q, R and S are points on the circumference of a circle.

TS is the tangent to the circle at the point S.

Angle RST = 35˚ and angle QRS = 101˚.

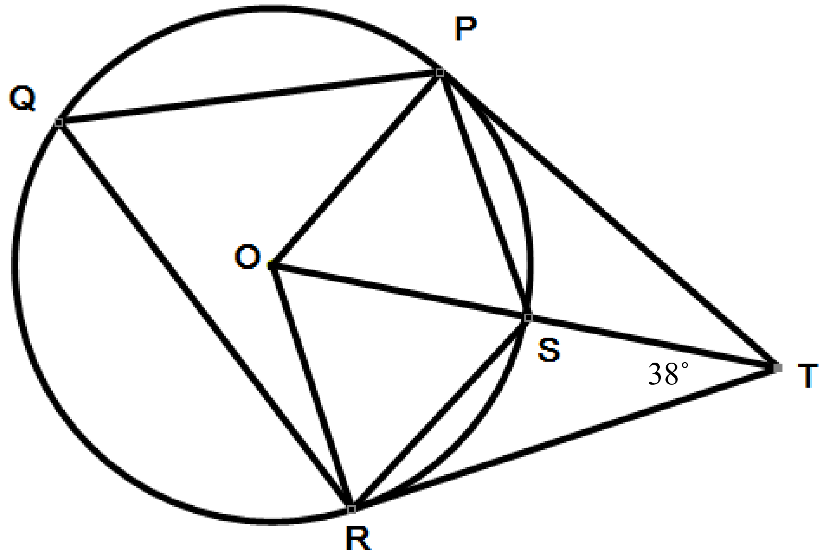


(a) Explain why QS cannot be a diameter of the circle.

(b) Find the size of angle:

(i) QPS (ii) SQR (iii) QPR

12.



P, Q, R and S are points on the circumference of a circle, centre, O.

PT and TR are tangents to the circle.

OST is a straight line.

Angle OTR = 38˚.

Find the size of the angle:

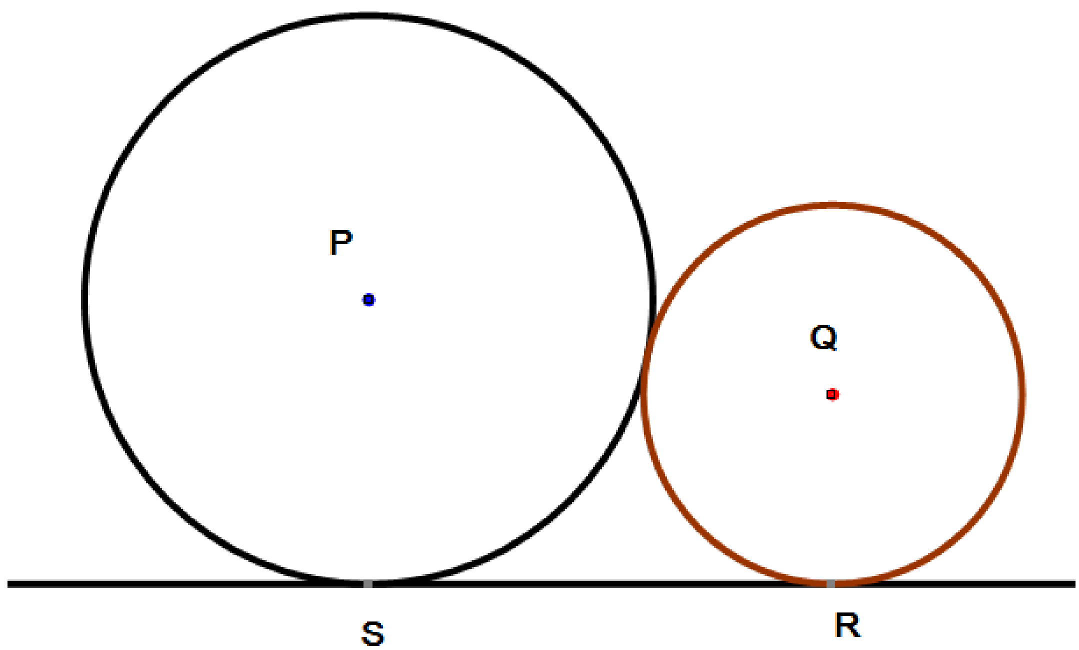
(i) ROT (ii) PQR (iii) SRT (iv) PSO (v) PST

13. The diagram shows a circle of radius 8cm, with centre P and a circle of radius 5cm with centre Q. The circles touch externally and have a common tangent RS.

(a) Explain why the quadrilateral PQRS is a trapezium.

(b) Calculate the length of RS, giving your answer in the form $m\sqrt{10}$, where m is a positive integer to be found.

* (c) If the circle with centre P has a radius of R cm and the circle with centre Q has a radius of r cm, show that the length of RS is given by $2\sqrt{Rr}$ cm.



14. P, Q and R are points on the circumference of a circle.

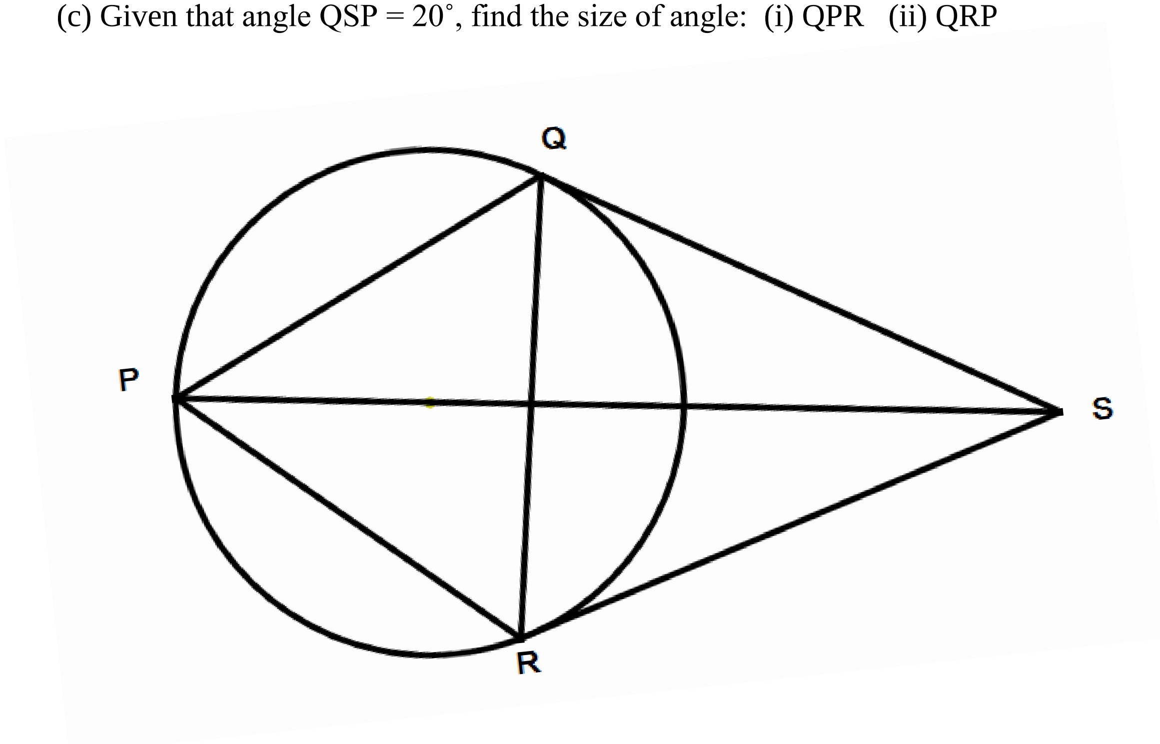
SQ and SR are tangents to the circle from a point S.

Angle PQR = Angle PRQ.

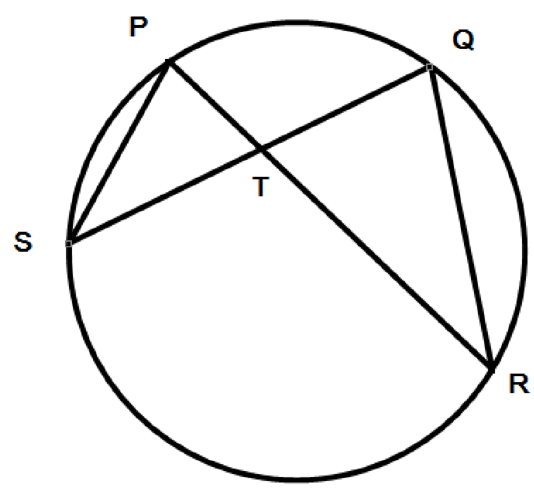
(a) Prove that triangles PQS and PRS are congruent.

(b) What type of quadrilateral is PRSR?

(c) Given that angle QSP = 20˚, find the size of angle: (i) QPR (ii) QRP



15. PR and QS are two chords of a circle that meet at the point T.



(a) Prove that triangles PTS and QTR are similar.

Given that PT = 3cm, TR = 8cm, and QT = 4cm,

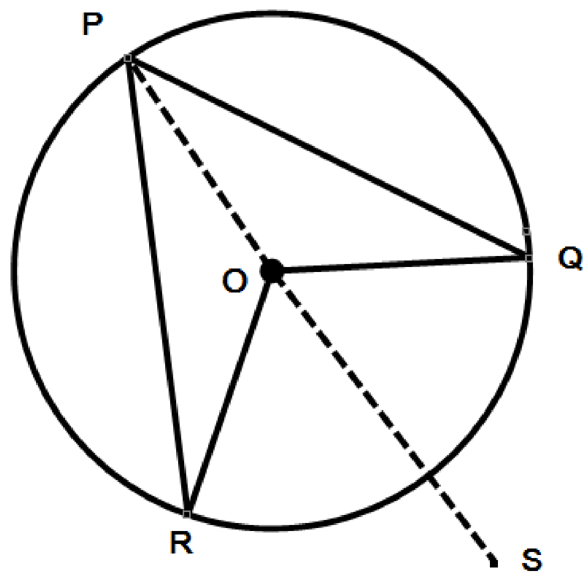
(b) Calculate the length of ST.

Some Proofs:

16. P, Q and R are points on the circumference of a circle, centre, O.

    The straight line POS has been drawn to help you.

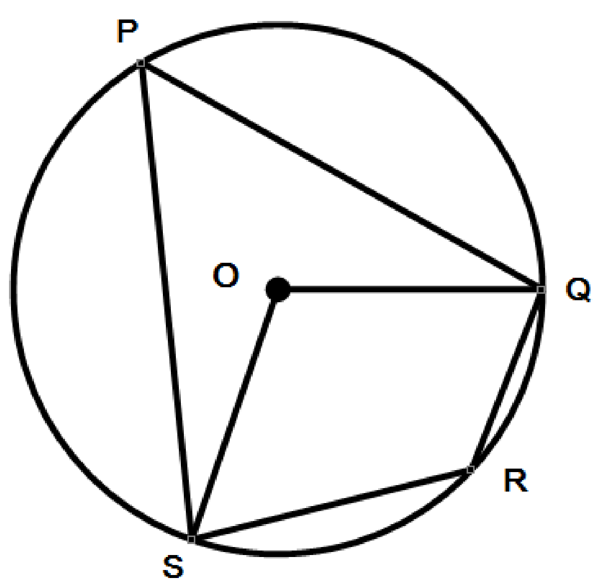
    Prove that Angle QOR is twice the size of angle QPR.



17. P, Q, R and S are points on the circumference of a circle, centre, O.

    Prove that angle SPQ + SRQ = 180˚

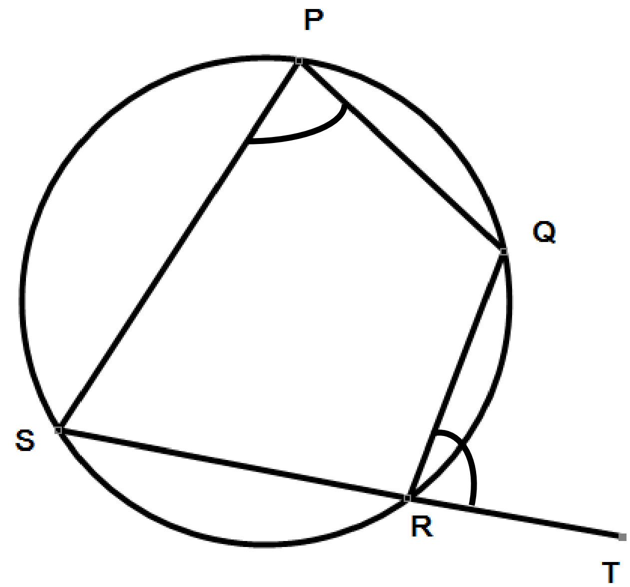
    (Opposite angles in a cyclic quadrilateral are supplementary).



18. P, Q and R are points on the circumference of a circle, centre, O.

The line SR is extended to T.

Prove that angle QRT = angle QPS.



Algebraic:

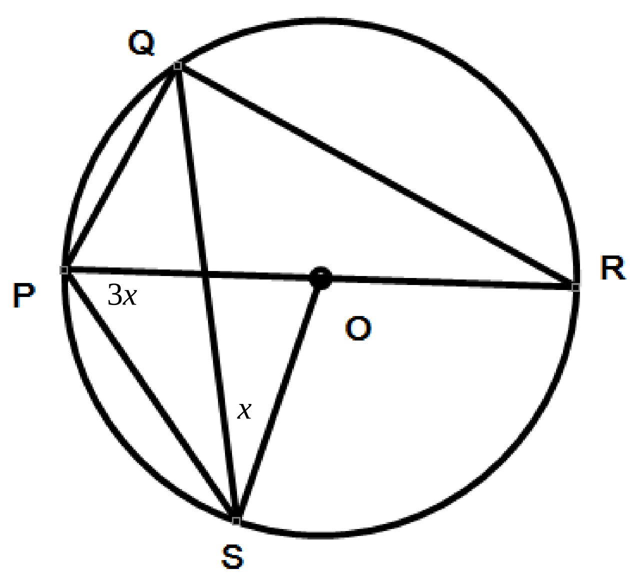
19. P, Q and R are points on the circumference of a circle, centre, O.

PR is a diameter of the circle. Angle QSO = x° and angle OPS = $3x^\circ$.

(a) Express in terms of x, the size of angle:
(i) SQR (ii) PQS (iii) PSQ (iv) SOP (v) PRQ (vi) QPR (vii) SOR

(b) Find the value of x if angle SOR = 120°.

(c) With this value of x, what type of triangle will OPS be?



Interesting and different!?

20. A, B, C, D and E are points on a circle.

Point C is due north of point D and point E is due west of point D.

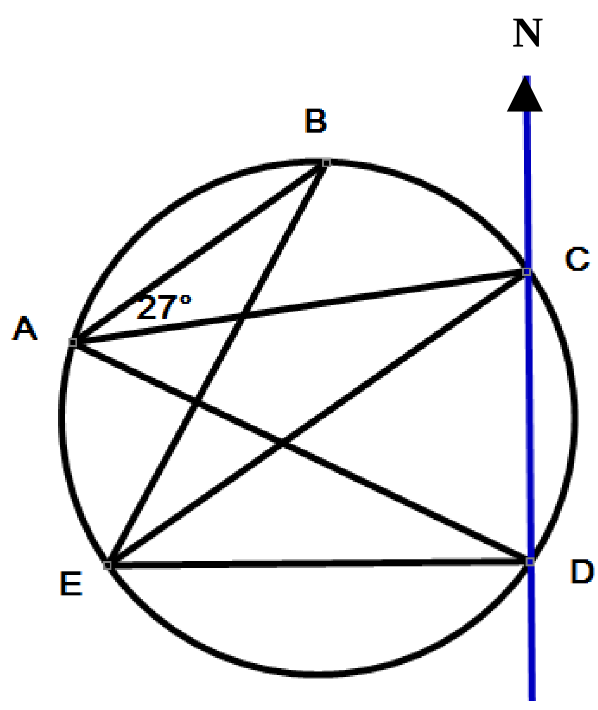
Angle CAB = 27˚.

The angle of elevation of point B from point E is 87˚.

(a) Find the size of the angle of elevation of point C from the point E.

(b) Why is EC a diameter of the circle?

(c) Find the size of the angle of elevation of point B from the point D.



Answers/Solutions

Only answers on this page. The Solutions are on the following pages. Please check answers.

Answers Only

(1) (a) 90° (b)(i) 10cm (ii) $24\,cm^2$ (iii) $A = 25\pi\,cm^2$
$C = 10\pi\,cm$

(c) (i) 60° (ii) 30°

(2) (i) 65° (ii) 44°

(3) (a) 32° (b) 58°

(4) (a) 37° (b) 74° (c) 16° (d) 53°

(5) (a) 90° (b) 48° (c) 96° (d) 42° (e) 84° (f) 42°

(6) (i) (a) 32° (b) 32° (c) 58° (d) 122°

(ii) $R\hat{Q}S = Q\hat{S}O = 32$ alternate angles

(iii) (a) 64° (b) 26°

(7) (a) 50° (b) 65° (c) 40° (d) 25° (e) 13°

(8) (i) 52° (ii) 26° (iii) 64° (iv) 26°

(9) (a)(i) 60° (ii) 30° (iii) 60° (iv) 120°

(b) Equilateral

(c) 50°

(10) (a) (i) 28° (ii) 28° (b) $62 + 28 = 90^\circ$

(11) (a) $101 \neq 90$

(b) (i) 79 (ii) 35° (iii) 35°

(12) (i) 52° (ii) 52° (iii) 26° (iv) 64° (v) 116°

(13) (b) $m = 4$ (c) Proof.

(14) (a) SSS (b) Kite (c)(i) 140° (ii) 70°

(15) (a) Proof (AAA) (b) $ST = 6\,cm$

(16), (17), (18) See solutions

(19) (i) $3x$ (ii) $90 - 3x$ (iii) $2x$ (iv) $180 - 6x$ (v) $2x$

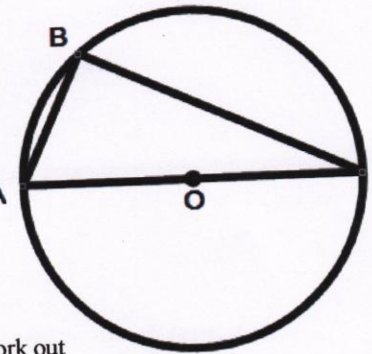
(vi) $90 - 2x$ (vii) $6x$ (b) $x = 20$ (c) Equilateral

(20) (a) 60° (c) 63°

Answers: (NOTE: Solutions not unique in some questions)

1. (a) A, B and C are points on the circumference of a circle, centre, O.
AC is the diameter of the circle.

Write down the size of angle ABC.



90° Angle in a semicircle is a right angle.

* (b) Given that AB = 6cm and BC = 8cm, work out

(i) the diameter of the circle,

Let $AC = x$ use Pythagoras' theorem
$x^2 = 6^2 + 8^2 = 36 + 64 = 100$
$x = 10$ cm Diameter = 10cm

(ii) the area of the triangle

$A = \frac{1}{2}bh = \frac{1}{2} \times 6 \times 8 = 24$ cm^2.

(iii) the area **and** circumference of the circle, **leaving your answer in terms of π.**

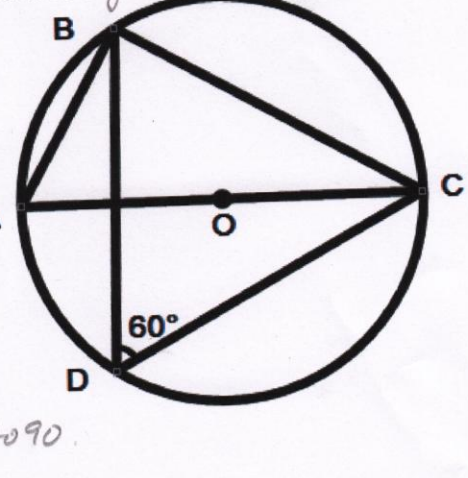
Radius, $r = 5$
$A = \pi r^2 = \pi \times 5^2$
$= 25\pi$ cm^2.

$C = \pi \times d$ OR $2\pi r$
$= \pi \times 10$
$= 10\pi$ cm

(c) D is a point on the circumference of the circle above such that angle BDC = 60°.

(i) Write down the size of angle CAB. (ii) Work out the size of angle ACB.

(i) $C\hat{A}B = 60^\circ$ Angles in the same segment are equal.
(ii) $A\hat{B}C = 90^\circ$ from (a)
$A\hat{C}B = 180 - (90 + 60)$
$= 180 - 150 = 30^\circ$
angles in a Δ add up to 180°.
(OR $A\hat{C}B = 90 - 60 = 30^\circ$)
If one angle is 90, the sum of the other two angles in Δ ABC add up to 90.



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2. P, Q, R and S are points on the circumference of a circle.

Angle PRS = 65° and angle QSR = 44°.

Find the size of angle (i) PQS (ii) QPR

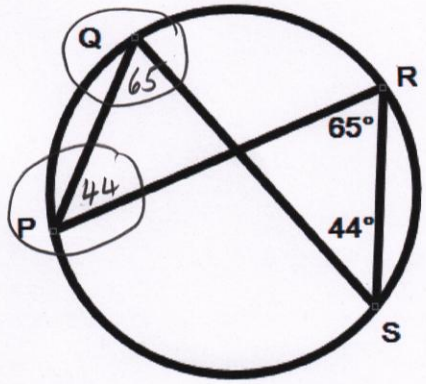
(i) $P\hat{Q}S = \underline{65^\circ}$

(ii) $P\hat{Q}R = \underline{44^\circ}$

Reason:

Angles in the same segment are equal

OR angles subtended by the same arc are equal



3. P, Q and R are points on the circumference of a circle, centre, O.
PR is the diameter of the circle and ST is a tangent to the circle at the point R.
Angle QRS = 58°.
(a) Work out the size of angle QRP.
(b) Work out the size of angle QPR.

(a) $Q\hat{R}P = 90^\circ - 58^\circ = \underline{32^\circ}$

Tangent and radius meet at 90°

$(P\hat{R}S = 90^\circ)$

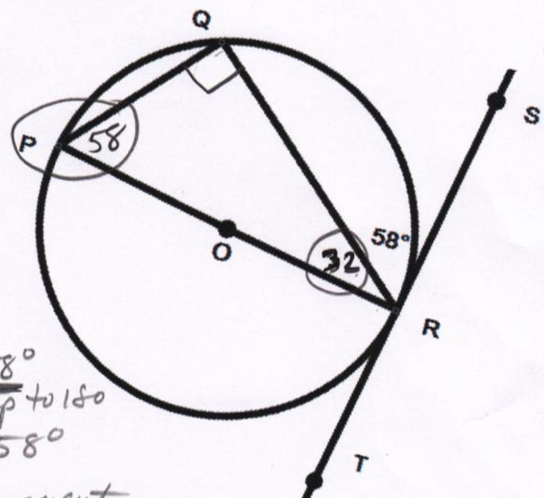
(b) $P\hat{Q}R = 90^\circ$ angle in a semicircle.

$Q\hat{P}R = 180 - (90 + 32)$

$= 180 - 122 = \underline{58^\circ}$

angles in a Δ add up to 180

OR $Q\hat{P}R = 90 - 32 = 58^\circ$



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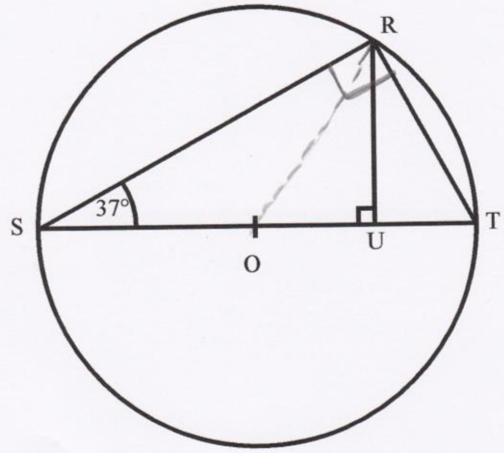
OR alternate segment theorem: $Q\hat{P}R = \underline{58^\circ}$

4.



R, S and T are points on the circumference of a circle, centre O.

ST is a diameter and Angle RST = 37°.

U is the point on ST such that angle RUS is a right angle.

(*a*) Work out the size of angle URT.

$S\hat{R}T = 90°$ angle in a semicircle

$S\hat{R}U = 90 - 37 = 53°$ angles in a Δ

$U\hat{R}T = 90 - 53 = \underline{37°}$

OR $R\hat{S}U + S\hat{R}U = 90°$

and $U\hat{R}T + S\hat{R}U = 90$

hence $U\hat{R}T = R\hat{S}U$

$= \underline{37°}$

(*b*) Work out the size of angle ROT.

$R\hat{O}T = 2 \times 37° = \underline{74°}$

angle at the centre = twice angle at the circumference (they must be subtended by the same arc OR in the same segment)

(*c*) Work out the size of angle ORU.

$O\hat{R}U = 90 - 74$

$= \underline{16°}$ ΔOUR is right-angled. and angles in a Δ add up to 180°

(d) Find the size of angle ORT.

ΔORT is isosceles ; OR = OT Radii

hence $O\hat{R}T = \frac{1}{2}(180° - 74°)$

$= \frac{1}{2}(106) = 53°$

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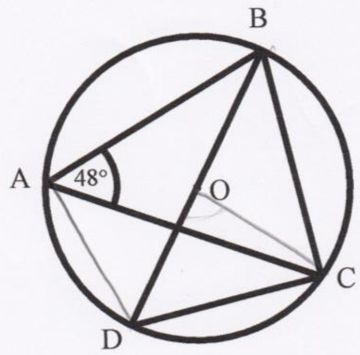
OR $O\hat{R}S = 37°$ ΔORS is isosceles.

and $S\hat{R}T = 90 \Rightarrow O\hat{R}T = 90 - 37 = \underline{53}$

5.



A, B, C and D are points on the circumference of a circle, centre O.

BD is a diameter of the circle.   Angle CAB = 48°.

(a) Write down the size of angle BCD. 90° angle in a semicircle $= 90^\circ$.

(b) Find the size of angle BDC. $B\hat{D}C = 48^\circ$ angles in the same segment are equal

(c) Find the size of angle BOC.

$B\hat{O}C = 2 \times 48 = 96^\circ$ angle at centre $= 2\times$ angle at circumference

(d) Find the size of angle CAD. $B\hat{A}D = 90^\circ$ angle in a semicircle.

$C\hat{A}D = 90 - 48^\circ = 42^\circ$

(e) Find the size of angle COD.

$C\hat{O}D = 2 \times C\hat{A}D = 2 \times 42 = 84^\circ$

angle at centre $= 2\times$ angle at circumference (same segment)

(f) Find the size of angle OCB.

$\triangle$ OCB is isosceles $OC = OB =$ radii

$B\hat{O}C = 96$ (c)

$180 - 96 = 84$ $O\hat{C}B = \frac{1}{2} \times 84 = 42^\circ$.

OR $O\hat{B}C = C\hat{A}D = 42^\circ$ (d)

and $O\hat{C}B = O\hat{B}C$ isosc. $\triangle$.

$\therefore O\hat{C}B = 42^\circ$

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6. P, Q, R and S are points on the circumference of a circle, centre, O.

TU is a tangent to the circle at the point S.

Angle ROS = 64° and angle QSU = 58°.

(i) Find the size of angle:

(a) OSQ

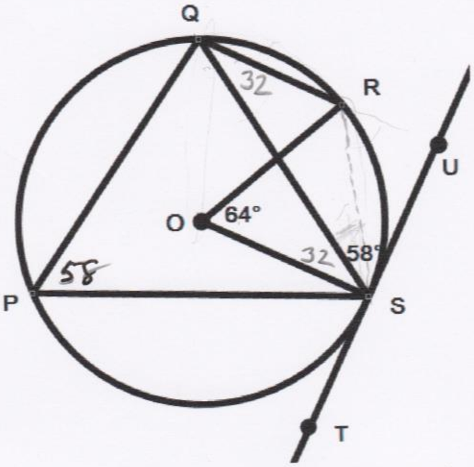
(b) SQR

(c) QPS

(d) QRS

(ii) Why are the lines QR and OS parallel?

(iii) Find the size of angle (a) QRO (b) QSR



(i) (a) $O\hat{S}Q = 90 - 58^\circ = \underline{\underline{32^\circ}}$ tangent and radius meet at 90°

(b) $S\hat{Q}R = \frac{1}{2} \times 64 = \underline{\underline{32^0}}$ angle at centre is twice the angle at the circumference (they must subtend the same arc)

(c) $Q\hat{P}S = \underline{\underline{58^0}}$ ~~alter~~

alternate segment theorem.

(angle between a tangent and a chord = angle in the alternate segment; they both subtend arc QS)

(d) PQRS is a cyclic quadrilateral

opposite angles are supplementary.

Hence $Q\hat{R}S = 180 - 58 = \underline{\underline{122^0}}$

(ii) $R\hat{Q}S = Q\hat{S}O = \underline{\underline{32}}$ alternate angles equal, hence QR is parallel to OS.

(iii) (a) $Q\hat{R}O = \underline{\underline{64^0}}$ alternate angles.

(b) $\triangle$ ORS is isosceles OR = OS radii

hence $O\hat{S}R = \frac{1}{2}(180 - 64)$

$= \frac{1}{2}(116) = 58^0$

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hence $Q\hat{S}R = 58 - 32 = \underline{\underline{26^0}}$

7. P, Q, R and S are points on the circumference of a circle, centre, O.

PST is a straight line.

PQ = PS

Angle SOQ = 100° and angle RST = 78°

Work out the size of angle:

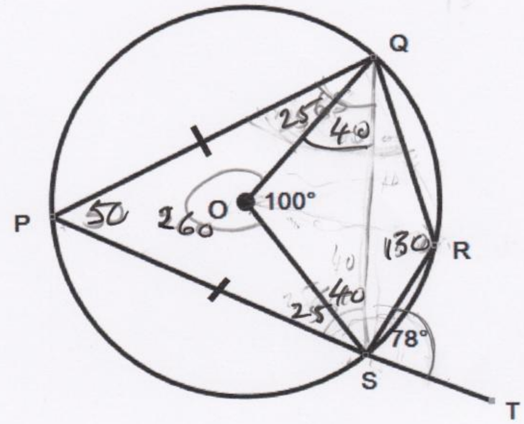
(a) QRS

(b) PQS

(c) OQS

(d) PSO

(e) SQR



(a) $Q\hat{P}S = \frac{1}{2} \times 100 = \underline{50^\circ}$ angle at centre = 2× angle at circumference.

$Q\hat{R}S = 180 - 50^\circ = \underline{130^\circ}$ opposite angles in a cyclic quadrilateral are supplementary.

OR Reflex $S\hat{O}Q = 360 - 100 = 260^\circ$

$Q\hat{R}S = \frac{1}{2} \times 260^\circ = \underline{130^\circ}$ angle at centre = 2× angle at circumference.

(b) $P\hat{Q}S = \frac{1}{2}(180 - 50) = \frac{1}{2}(130) = \underline{65^\circ}$

$\triangle PQS$ is isosceles (given).

(c) $O\hat{Q}S = \frac{1}{2}(180 - 100) = \frac{1}{2} \times 80 = \underline{40^\circ}$ $\triangle OQS$ is isosceles.

(d) $P\hat{S}O = 65 - 40 = \underline{25^\circ}$

(e) $P\hat{Q}S = 78^\circ$ exterior angle of a cyclic quadrilateral = opp interior angle

But $P\hat{Q}S = 65^\circ$ (b)

OR $P\hat{S}R = 180 - 78 = 102^\circ$

$P\hat{Q}R = 180 - 102 = 78^\circ$ opp. angles in a cyclic quad. are supplementary.

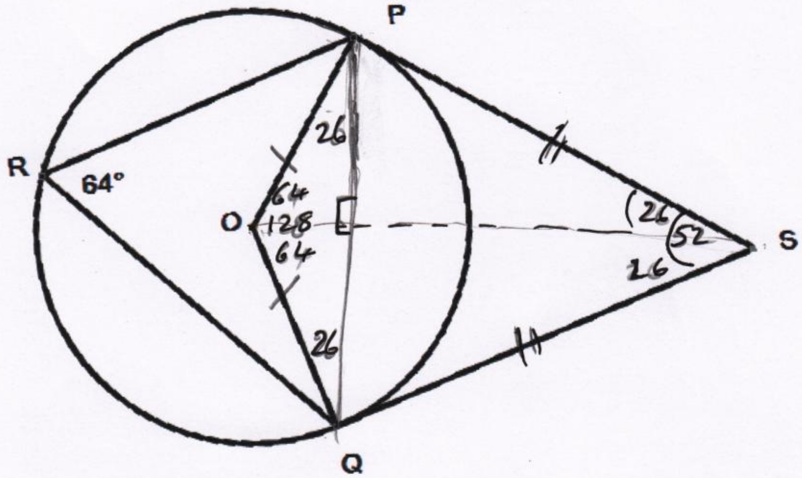
hence $S\hat{Q}R = 78 - 65$

$= \underline{13^\circ}$

8. P, Q and R are points on the circumference of a circle, centre, O.

Angle PRQ = 64°. SP and SQ are tangents to the circle at the points P and Q respectively.

Work out the size of angle (i) PSQ (ii) PQO (iii) POS (iv) QSO



(i) $P\hat{O}Q = 2 \times 64 = \underline{128^\circ}$ angle at Centre = 2× angle at Circumference

$O\hat{P}S = 90^\circ = O\hat{Q}S$ tangent radius meet at 90°

hence $P\hat{S}Q = 180 - 128^\circ$ $(360 - (90 + 90 + 128))$

$= \underline{\underline{52^\circ}}$ angles in a quadrilateral add up to 360°

(ii) $P\hat{Q}O = \frac{1}{2}(180 - 128^\circ) = \frac{1}{2} \times 52 = \underline{\underline{26^\circ}}$ $\triangle OPQ$ is isosceles $OP = OQ$ radii

(iii) $P\hat{O}S = \frac{1}{2} \times 128 = \underline{\underline{64^\circ}}$ Symmetry of the kite OPSQ

(iv) $Q\hat{S}O = \frac{1}{2} \times 52 = \underline{\underline{26^\circ}}$ $PS = PQ$ tangents from a point

9. P, Q and R are points on the circumference of a circle, centre, O.

Angle PSQ = 60°. SP and SQ are tangents to the circle at the points P and Q respectively.

(a) Work out the size of angle:

(i) QPS

(ii) PQO

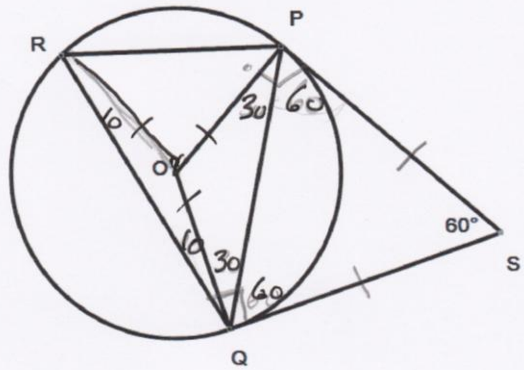
(iii) PRQ

(iv) POQ

(b) What type of triangle is PQS?

(c) Given that angle OQR = 10°,

work out the size of angle OPR.



(a)(i) $Q\hat{P}S = \frac{1}{2}(180-60) = \frac{1}{2}\times 120 = 60^\circ$ PS = QS tangents from a point are equal

(ii) $P\hat{Q}O = 90-60 = 30^\circ$ $O\hat{Q}S = 90$ tangent radius and ΔOPQ is isosceles $OP = OQ$ = radii

(iii) $P\hat{R}Q = \frac{1}{2}\times P\hat{O}Q$ $P\hat{O}Q = 180-60 = 120^\circ$ Sum of angles in quadrilateral PSQO = 360°

$= \frac{1}{2}\times 120^\circ = 60^\circ$

OR angle in alternate segment theorem: $P\hat{R}Q = Q\hat{P}S = 60^\circ$

(iv) $P\hat{O}Q = 120^\circ$ see (iii)

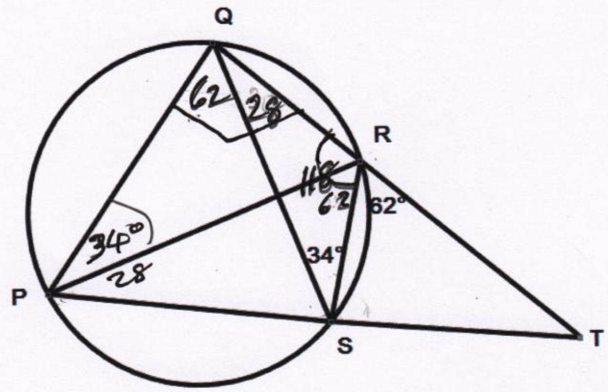
(b) Equilateral triangle

(c) ΔORQ is isosceles $OR = OQ$ radii

$\therefore O\hat{R}Q = 10^\circ$ $O\hat{R}P = 60-10 = 50^\circ$

$O\hat{P}R = O\hat{R}P = 50^\circ$ ΔOPR is isosceles with $OR = OP$

10.



P, Q, R and S are points on the circumference of a circle.

PST and QRT are straight lines.

Angle QSR = 34° and angle SRT = 62°.

(a) Find the size of the angle:

(i) SQR (ii) RPS

(b) Given that angle PRS = 62°, show that PR is a diameter of the circle.

(a)(i) $SRQ = 180 - 162 = 118°$ straight line

$SQR = 180 - (118 + 34)$ angles in a Δ add up to 180.

$= 180 - 152 = \underline{\underline{28°}}$

or $S\hat{Q}R = 62 - 34 = \underline{\underline{28°}}$ exterior angle (62) = sum of opp. interior angles.

(ii) $R\hat{P}S = S\hat{Q}R = \underline{\underline{28}}$ angles in the same segment.

(b) $P\hat{R}S = 62$ given

$P\hat{Q}S = \underline{\underline{62°}}$ angles in the same segment.

hence $P\hat{Q}R = 62 + 28 = 90°$

PR is a diameter. Angle in a semicircle $= 90°$

11. P, Q, R and S are points on the circumference of a circle.

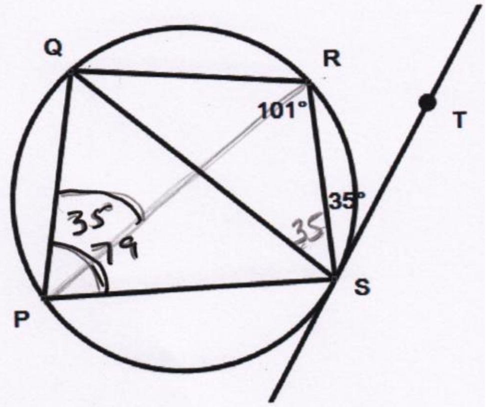
TS is the tangent to the circle at the point S.

Angle RST = 35° and angle QRS = 101°.

(a) Explain why QS cannot be a diameter of the circle.

(b) Find the size of angle:

(i) QPS (ii) SQR (iii) QPR



(a) $Q\hat{R}S = 101° \neq 90°$ (angle in a semicircle = 90°)

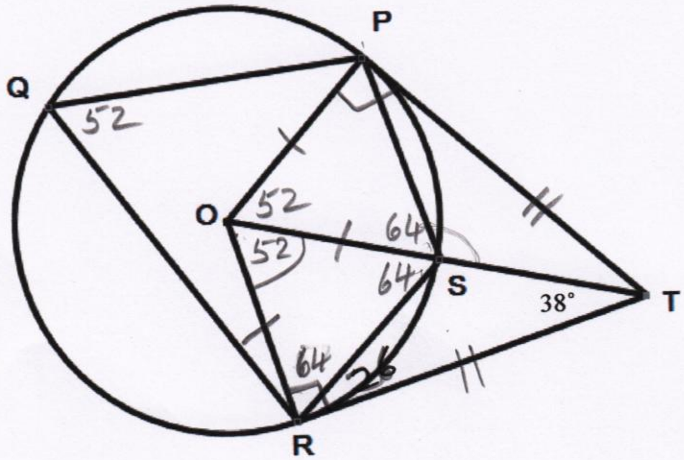
(b) (i) $Q\hat{P}S = 180 - 101°$

$= \underline{\underline{79°}}$ opposite angles in a cyclic quad. are supplementary.

(ii) $S\hat{Q}R = \underline{\underline{35°}}$ alternate segment theorem

(iii) $Q\hat{P}R = Q\hat{S}R = \underline{\underline{35°}}$ angles in the same segment.

12.



P, Q, R and S are points on the circumference of a circle, centre, O.

PT and TR are tangents to the circle.

OST is a straight line.

Angle OTR = 38°.

Find the size of the angle:

(i) ROT (ii) PQR (iii) SRT (iv) PSO (v) PST

(i) $O\hat{R}T = 90^\circ$ tangent radius at 90°

$R\hat{O}T = 90 - 38^\circ = \underline{52^\circ}$ angles in a Δ

(ii) $P\hat{Q}R = \frac{1}{2} \times P\hat{O}R = \frac{1}{2} \times (52 + 52)$ OPTR is a kite

$= \underline{52^\circ}$

PT = TR

OP = OR

(iii) $O\hat{R}S = \frac{1}{2}(180 - 52)$ Δ ORS is isosceles.

$= \frac{1}{2}(128) = \underline{64^\circ}$

$S\hat{R}T = 90 - 64 = \underline{26^\circ}$ tangent radius meet at 90°

(iv) $P\hat{S}O = \frac{1}{2}(180 - 52)$ OSP is an isosceles Δ

$= \frac{1}{2} \times 128 = \underline{64}$

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(v) $P\hat{S}T = 180 - 64$

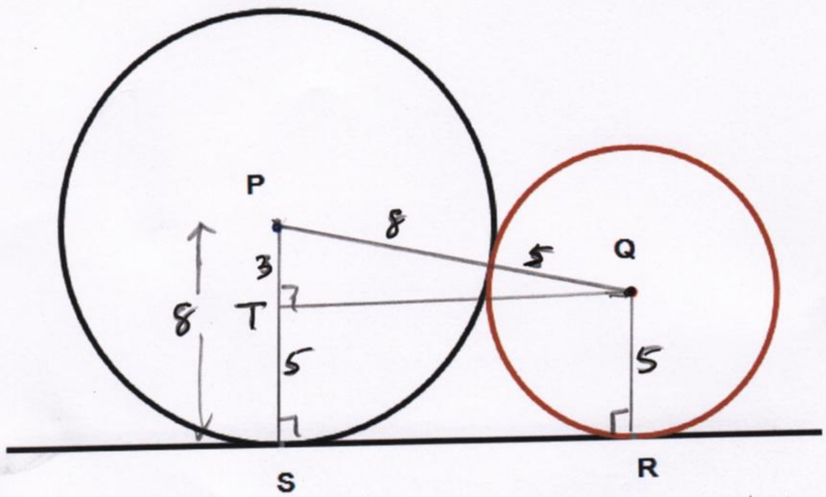
$= \underline{116^\circ}$ OST is a straight line

13. The diagram shows a circle of radius 8cm, with centre P and a circle of radius 5cm with centre Q. The circles touch externally and have a common tangent RS.

(a) Explain why the quadrilateral PQRS is a trapezium.

(b) Calculate the length of RS, giving your answer in the form $m\sqrt{10}$, where m is a positive integer to be found.

* (c) If the circle with centre P has a radius of R cm and the circle with centre Q has a radius of r cm, show that the length of RS is given by $2\sqrt{Rr}$ cm.



(a) tangent and radius meet at 90°, hence both PS and QR are perpendicular to SR and hence parallel. Hence PQRS is a trapezium.

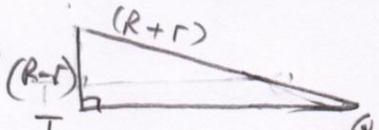
(b) $PQ = 13 \quad PT = 8-5 = 3 \quad QT^2 = 13^2 - 3^2 = 169 - 9$

$= 160 = 16 \times 10$

$QT = \sqrt{16 \times 10} = 4\sqrt{10}$

$SR = QT = 4\sqrt{10} \quad \underline{\underline{m = 4}}$

(c)



$TQ^2 = (R+r)^2 - (R-r)^2$

$= R^2 + 2Rr + r^2 - (R^2 - 2Rr + r^2)$

$= R^2 + 2Rr + r^2 - R^2 + 2Rr - r^2$

$TQ = \sqrt{4Rr} = \underline{\underline{2\sqrt{Rr}}} \quad TQ = SR$

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14. P, Q and R are points on the circumference of a circle.

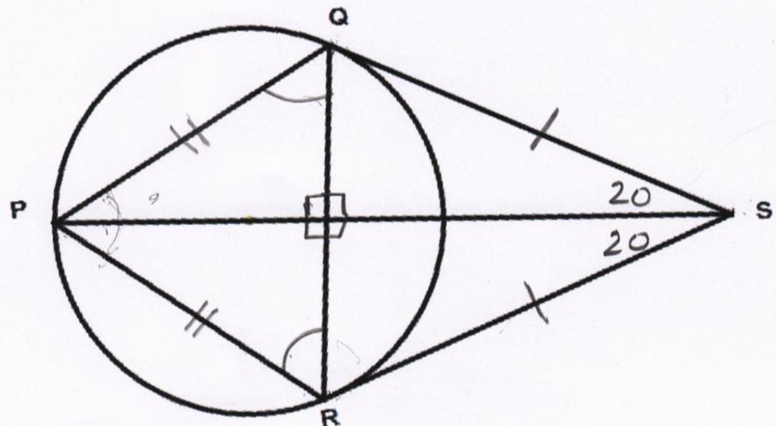
SQ and SR are tangents to the circle from a point S.

Angle PQR = Angle PRQ.

(a) Prove that triangles PQS and PRS are congruent.

(b) What type of quadrilateral is PRSR?

(c) Given that angle QSP = 20°, find the size of angle: (i) QPR (ii) QRP



(a) ΔPQS ΔPRS

$PQ = PR$ ΔPQR given isosceles.

$SQ = SR$ tangents from a point to a circle are equal.

$PS = PS$ common to both triangles

By SSS the triangles are congruent.

(b) PRSR is a kite ($PQ = PR$, $SQ = SR$)

(c) (i) QR is perpendicular to PS (properties of a kite)

hence $RQS = 90 - 20 = 70°$ angles in a Δ add up to 180.

$Q\hat{P}S = RQS = 70°$ alternate segment theorem.

and $Q\hat{P}S = Q\hat{P}R$ congruent triangles

hence $Q\hat{P}R = 70 + 70 = 140°$

(ii) $Q\hat{R}P = R\hat{Q}S = 70°$ ΔSRQ is isosceles.

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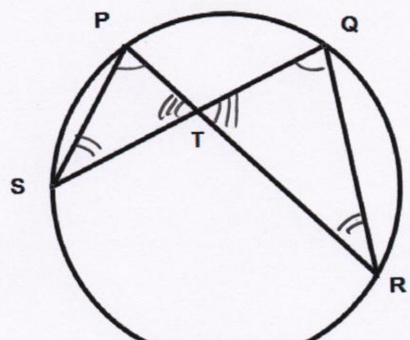
15. PR and QS are two chords of a circle that meet at the point T.

(a) Prove that triangles PTS and QTR are similar.

Given that PT = 3cm, TR = 8cm, and QT = 4cm,

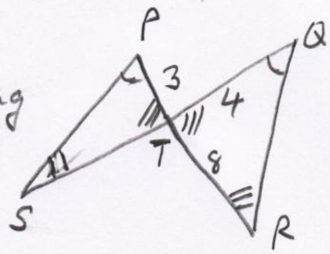
(b) Calculate the length of ST.



(a) $S\hat{P}T = T\hat{Q}R$
$P\hat{S}T = T\hat{R}Q$ } angles in the same segment are equal (they are subtended by the same arc)
$P\hat{T}S = Q\hat{T}R$ → Vertically opposite angles

hence the triangles are equiangular
hence by AAA the triangles are similar.

(b) Since the triangles are similar, the ratio of the corresponding sides are equal.



$$\frac{ST}{8} = \frac{3}{4}$$

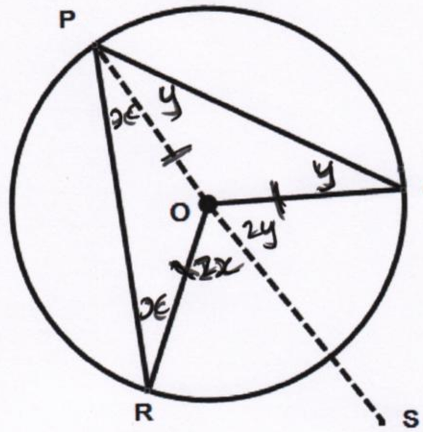
$$ST = 8 \times \frac{3}{4} = \underline{\underline{6\text{ cm}}}$$

Some Proofs:

16. P, Q and R are points on the circumference of a circle, centre, O.

The straight line POS has been drawn to help you.

Prove that Angle QOR is twice the size of angle QPR.



Let $R\hat{P}O = x$

hence $P\hat{R}O = x$

$\triangle OPR$ is isosceles

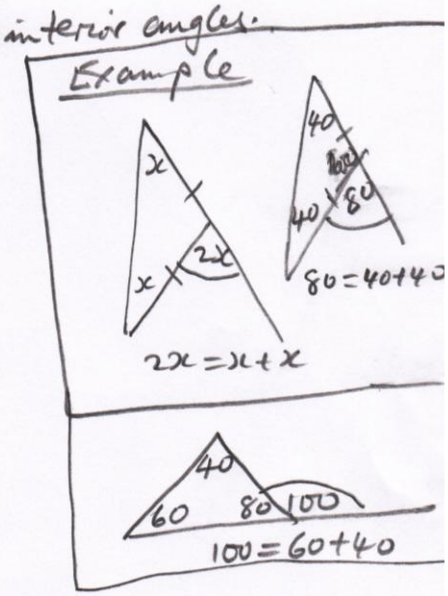
$OP = OR$ radii

Let $O\hat{P}Q = y$

hence $O\hat{Q}P = y$

Same reason as for x.

Now: the exterior angle of a triangle = the sum of the 2 opposite interior angles.



hence

$R\hat{O}S = 2x$

and $Q\hat{O}S = 2y$

Now $R\hat{O}Q = 2x + 2y$

$R\hat{O}Q = 2(x+y)$

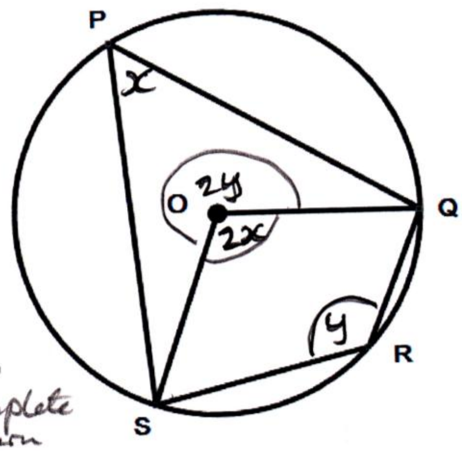
$R\hat{O}Q = 2 \times Q\hat{P}R$

Note: other methods of proofs may exist.

17. P, Q, R and S are points on the circumference of a circle, centre, O.

Prove that angle SPQ + SRQ = 180°

(Opposite angles in a cyclic quadrilateral are supplementary).



Let $S\hat{P}Q = x$

and $S\hat{R}Q = y$

then

$S\hat{O}Q = 2x$ (obtuse)

Reflex $S\hat{O}Q = 2y$ (Reflex)

Reason: angle at the centre = 2×angle at circumference.

but $2x + 2y = 360°$ complete turn

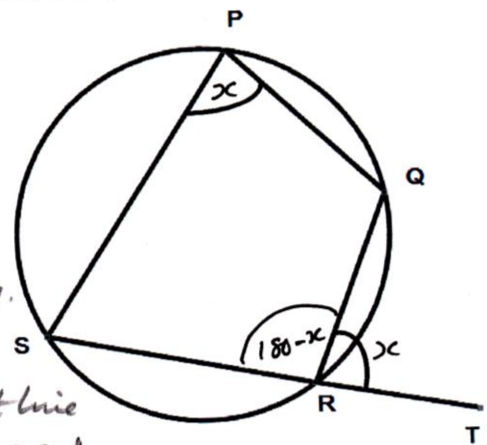
hence $x + y = 180°$

$\therefore$ $S\hat{P}Q + S\hat{R}Q = 180$

18. P, Q and R are points on the circumference of a circle, centre, O.

The line SR is extended to T.

Prove that angle QRT = angle QPS.



Let $S\hat{P}Q = x$

$\Rightarrow$ $S\hat{R}Q = 180 - x$ opposite angles in a cyclic quad. are supplementary. (add up to 180)

But SRT is a straight line

hence $S\hat{R}Q + Q\hat{R}T = 180°$

$180 - x + Q\hat{R}T = 180°$

$Q\hat{R}T = 180 - 180 + x$

$Q\hat{R}T = x$

hence $S\hat{P}Q = Q\hat{R}T$

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Algebraic:

19. P, Q and R are points on the circumference of a circle, centre, O.

PR is a diameter of the circle. Angle QSO = $x°$ and angle OPS = $3x°$.

(a) Express in terms of x, the size of angle:
(i) SQR (ii) PQS (iii) PSQ (iv) SOP (v) PRQ (vi) QPR (vii) SOR

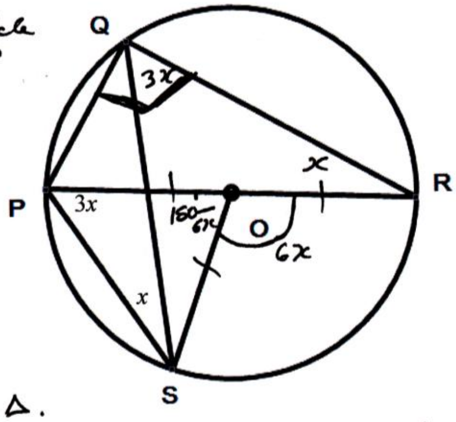
(b) Find the value of x if angle SOR = 120°.

(c) With this value of x, what type of triangle will OPS be?

(a)(i) $S\hat{Q}R = 3x$ angles in the same segment.

(ii) $P\hat{Q}R = 90$ angle in a semicircle $= 90°$

$\therefore P\hat{Q}S = 90 - 3x$ angles in a Δ.



(iii) ΔOPQ is isosceles
$OP = OS$ Radii

hence
$O\hat{P}S = O\hat{S}P = 3x$

hence $P\hat{S}Q = 3x - x$
$= 2x$

(iv) $S\hat{O}P = 180 - 6x$ angles in a Δ.

(v) $P\hat{R}Q = P\hat{S}Q = 2x$ angles in the same segment

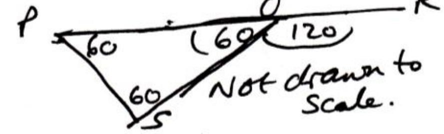
(vi) $Q\hat{P}R = 90 - 2x$ angles in a Δ.

(vii) $S\hat{O}R = 2 \times S\hat{Q}R$ angle at Centre $= 2 \times$ angle at circumference.
$= 2 \times 3x$
$S\hat{O}R = 6x$

(b) $S\hat{O}R = 6x = 120°$
$x = 20°$

(c) Equilateral triangle

$3x = 60°$, $180 - 6x = 60$
$2x = 40$
$x = 20$



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Interesting and different!?

20. A, B, C, D and E are points on a circle.

Point C is due north of point D and point E is due west of point D.

Angle CAB = 27˚.

The angle of elevation of point B from point E is 87˚.

(a) Find the size of the angle of elevation of point C from the point E.

(b) Why is EC a diameter of the circle?

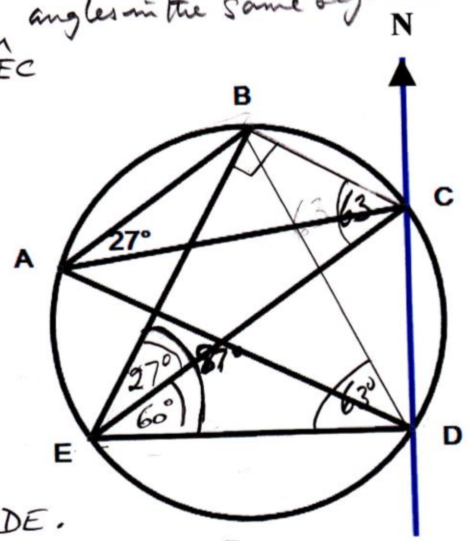
(c) Find the size of the angle of elevation of point B from the point D.

(a) $B\hat{E}C = B\hat{A}C = 27^\circ$ angles in the same segment

$\therefore C\hat{E}D = B\hat{E}D - B\hat{E}C$

$= 87 - 27$

$= \underline{60^\circ}$



(b) C is north of D
and E is west of D
hence $C\hat{D}E = 90^\circ$
hence EC is a diameter.
(angle in a semicircle $= 90^\circ$)

(c) The required angle is BDE.

Now $E\hat{B}C = 90^\circ$, angle in a semicircle.

in Δ EBC, $B\hat{E}C = 27^\circ$

hence $E\hat{C}B = 90 - 27 = 63^\circ$. (angles in a Δ)

But $B\hat{D}E = B\hat{C}E$ angles in the same segment.

hence $B\hat{D}E = \underline{63^\circ}$

I hope you find this useful and challenging for some students.

Answers need checking, please and let me know if you find any errors.

Thank you.