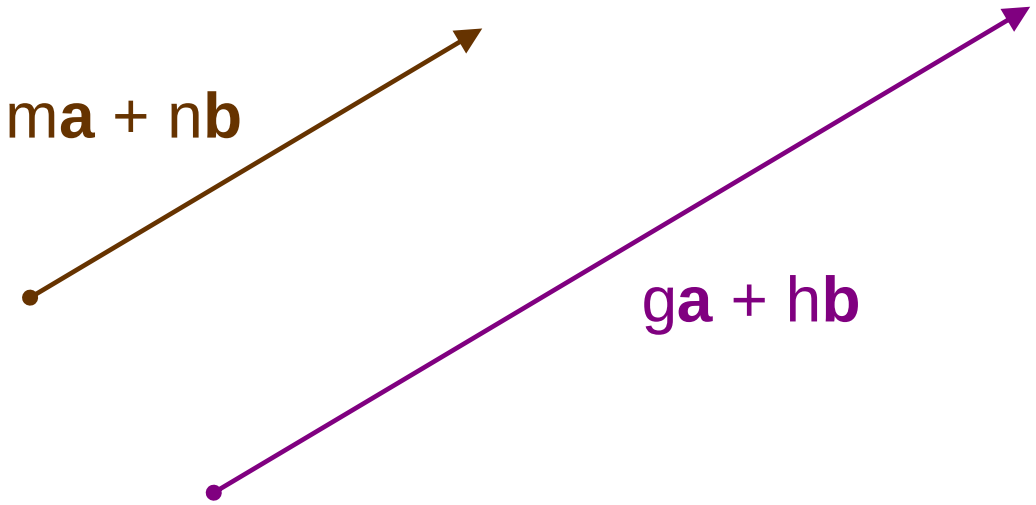
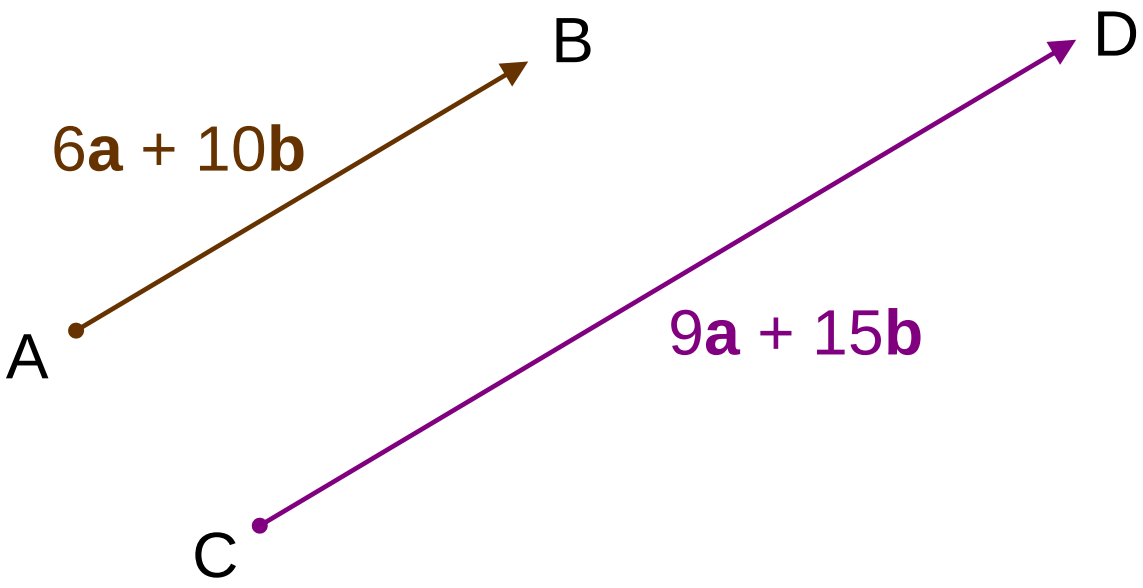
if two vectors are parallel
(or are sections of the same line)



then $\frac{g}{m} = \frac{h}{n}$

if two vectors are parallel
(or sections of the same line)



$$\frac{9}{6} = \frac{15}{10} = 1\frac{1}{2}$$

$$\overrightarrow{CD} = 1\frac{1}{2}\overrightarrow{AB}$$

$$\overrightarrow{AB} = \frac{2}{3}\overrightarrow{CD}$$

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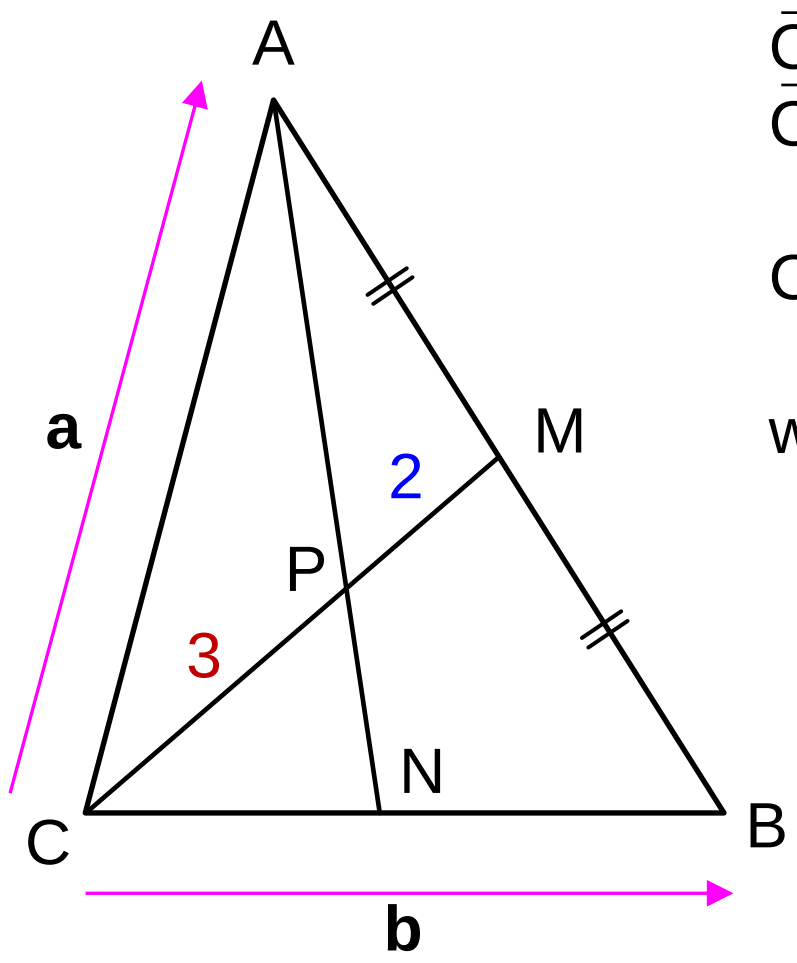


M = midpoint of AB

$\overrightarrow{CA}$ = **a**
$\overrightarrow{CB}$ = **b**

CP : PM = 3 : 2

work out the ratio CN : NB

method (i)

$\overrightarrow{AB} =$

$\overrightarrow{AM} =$

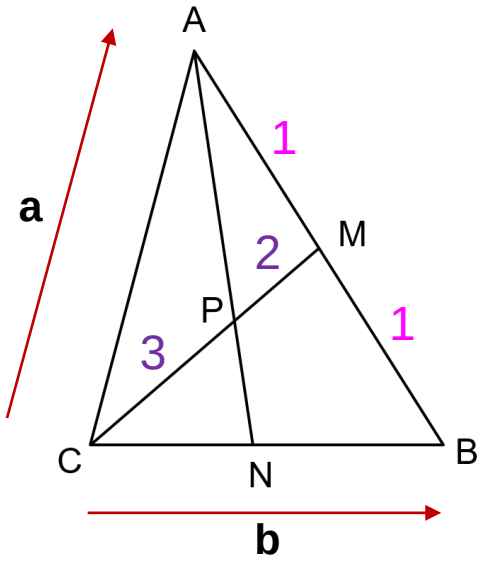
$\overrightarrow{CM} =$

$\overrightarrow{CP} =$

$\overrightarrow{AP} =$

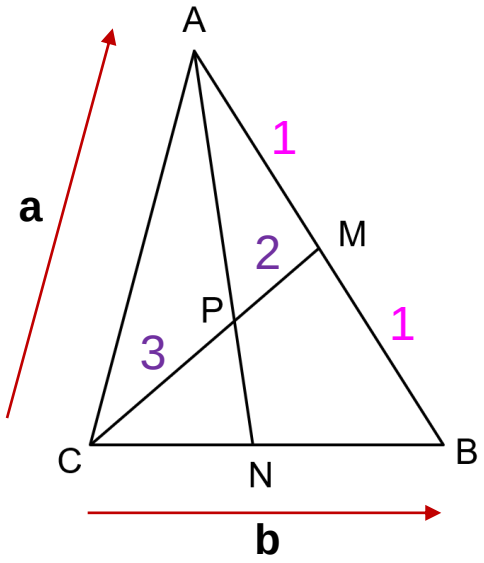
$\overrightarrow{CN} = k\,\overrightarrow{CB}$

$\overrightarrow{AN} =$



$\overrightarrow{AN}$ is a multiple of $\overrightarrow{AP}$
since in the same direction

method (i)



$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$

$\overrightarrow{AM} = \frac{1}{2}\overrightarrow{AB} = -\frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$\overrightarrow{CM} = \overrightarrow{CA} + \overrightarrow{AM} = \mathbf{a} - \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$= \frac{1}{2}\mathbf{a} + \frac{1}{2}\mathbf{b}$

$\overrightarrow{CP} = \frac{3}{5}\overrightarrow{CM} = \frac{3}{10}\mathbf{a} + \frac{3}{10}\mathbf{b}$

$\overrightarrow{AP} = \overrightarrow{AC} + \overrightarrow{CP} = -\mathbf{a} + \frac{3}{10}\mathbf{a} + \frac{3}{10}\mathbf{b}$

$= -\frac{7}{10}\mathbf{a} + \frac{3}{10}\mathbf{b}$

$\overrightarrow{CN} = k\overrightarrow{CB}$

$\overrightarrow{AN} = \overrightarrow{AC} + \overrightarrow{CN} = -\mathbf{a} + k\mathbf{b}$

$\overrightarrow{AN}$ is a multiple of $\overrightarrow{AP}$
since in the same direction

$-\frac{7}{10} \times \frac{10}{7} = -1$ so $k = \frac{3}{10} \times \frac{10}{7} = \frac{3}{7}$

method (ii)

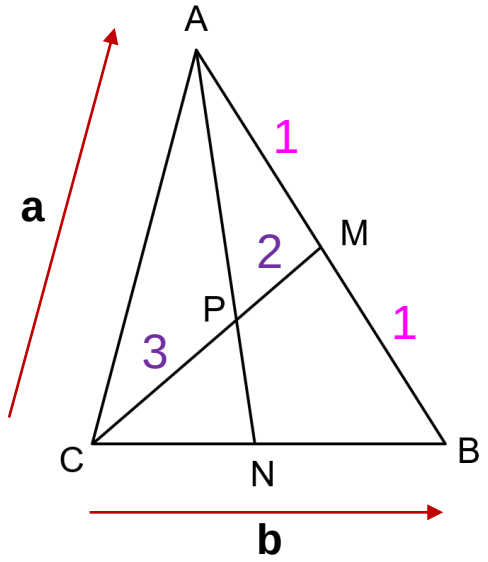
$\overrightarrow{CM} = \frac{1}{2}\,\mathbf{a} + \frac{1}{2}\,\mathbf{b}$

$\overrightarrow{CP} = \frac{3}{5}\,\overrightarrow{CM} = \frac{3}{10}\,\mathbf{a} + \frac{3}{10}\,\mathbf{b}$

$\overrightarrow{AN} = \overrightarrow{AC} + \overrightarrow{CN} = -\mathbf{a} + k\,\mathbf{b}$

$\overrightarrow{PN} = \overrightarrow{PC} + \overrightarrow{CN} = -\frac{3}{10}\,\mathbf{a} - \frac{3}{10}\,\mathbf{b} + k\,\mathbf{b}$

$= -\frac{3}{10}\,\mathbf{a} + (k - \frac{3}{10})\,\mathbf{b}$



$\overrightarrow{AN}$ is a multiple of $\overrightarrow{PN}$
since in the same direction

$$\frac{k - 3/10}{k} = \frac{-3/10}{-1}$$

$$7/10\ k = 3/10$$

$$k = 3/7$$

method (iii)

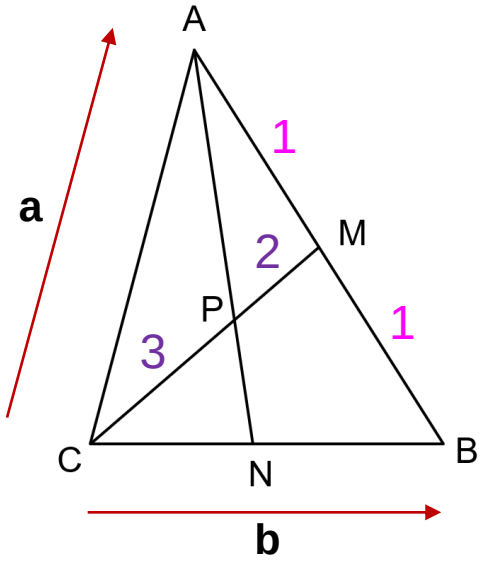
$\overrightarrow{CM} = \frac{1}{2}\,\mathbf{a} + \frac{1}{2}\,\mathbf{b}$

$\overrightarrow{CP} = \frac{3}{5}\,\overrightarrow{CM} = \frac{3}{10}\,\mathbf{a} + \frac{3}{10}\,\mathbf{b}$

$\overrightarrow{AP} = -\frac{7}{10}\,\mathbf{a} + \frac{3}{10}\,\mathbf{b}$

$\overrightarrow{AN} = f\,\overrightarrow{AP} = -\frac{7}{10}f\,\mathbf{a} + \frac{3}{10}f\,\mathbf{b}$

$\overrightarrow{CN} = \overrightarrow{CA} + \overrightarrow{AN} = \mathbf{a} - \frac{7}{10}f\,\mathbf{a} + \frac{3}{10}f\,\mathbf{b}$

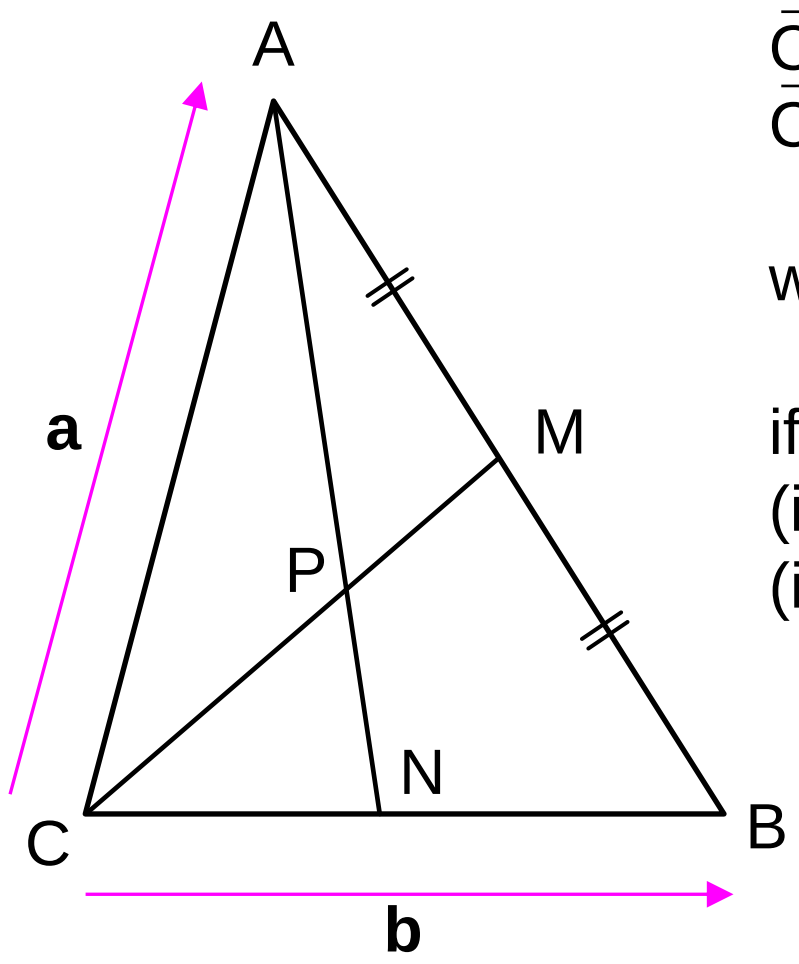


$\overrightarrow{CN}$ is a multiple of $\overrightarrow{CB}$ (= **b**)
since in the same direction
so the coefficient of **a** for $\overrightarrow{CN}$
must be zero

$1 - \frac{7}{10}f = 0$

$f = \frac{10}{7}$

$\overrightarrow{CN} = \frac{3}{7}\,\mathbf{b}$



M = midpoint of AB

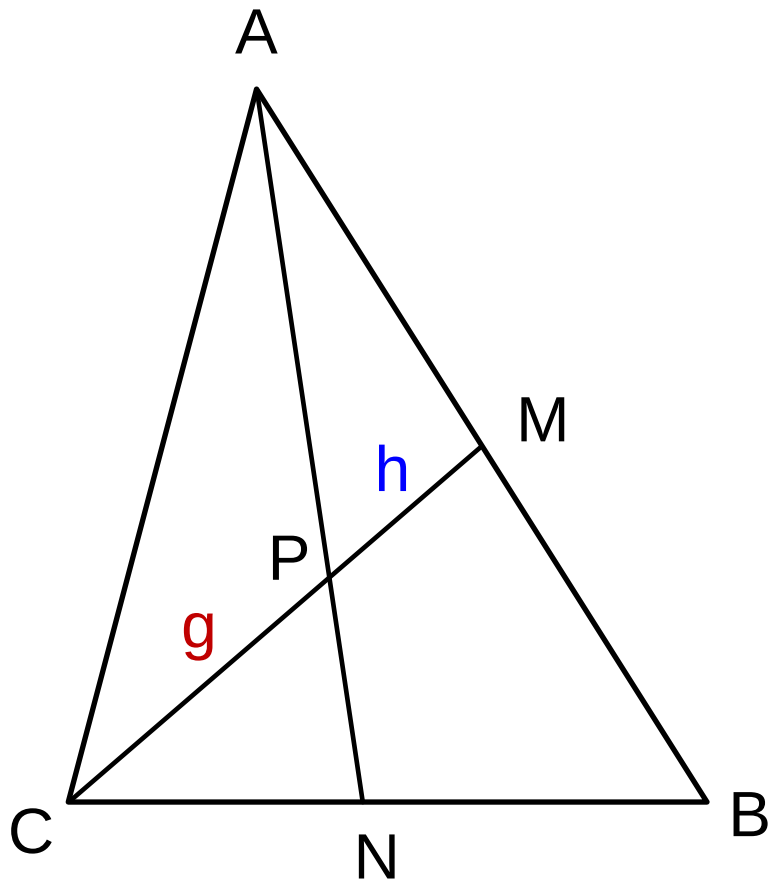
$\overrightarrow{CA}$ = **a**
$\overrightarrow{CB}$ = **b**

work out the ratio CN : NB

if
(i) CP : PM = 2 : 1
(ii) CP : PM = 3 : 1

(i) 1 : 1
(ii) 3 : 2

what if CP : PM = g : h ?



M is the midpoint of AB

$\overrightarrow{CA} = \mathbf{a}$

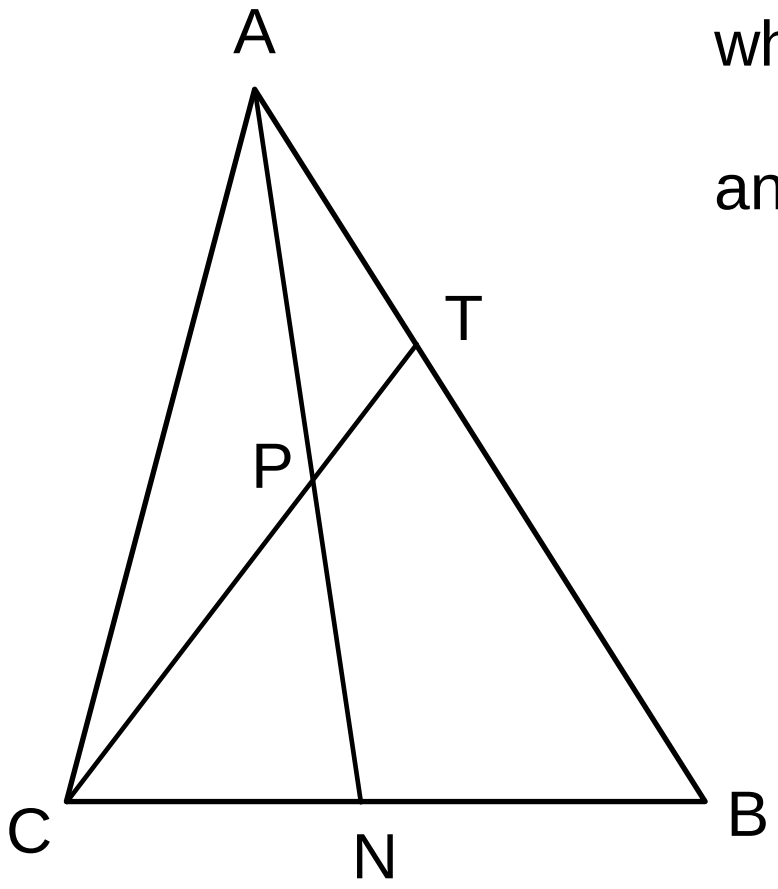
$\overrightarrow{CB} = \mathbf{b}$

CP : PM = g : h

CN : NB = g : 2h

and NP : PA = g : (2h + g)

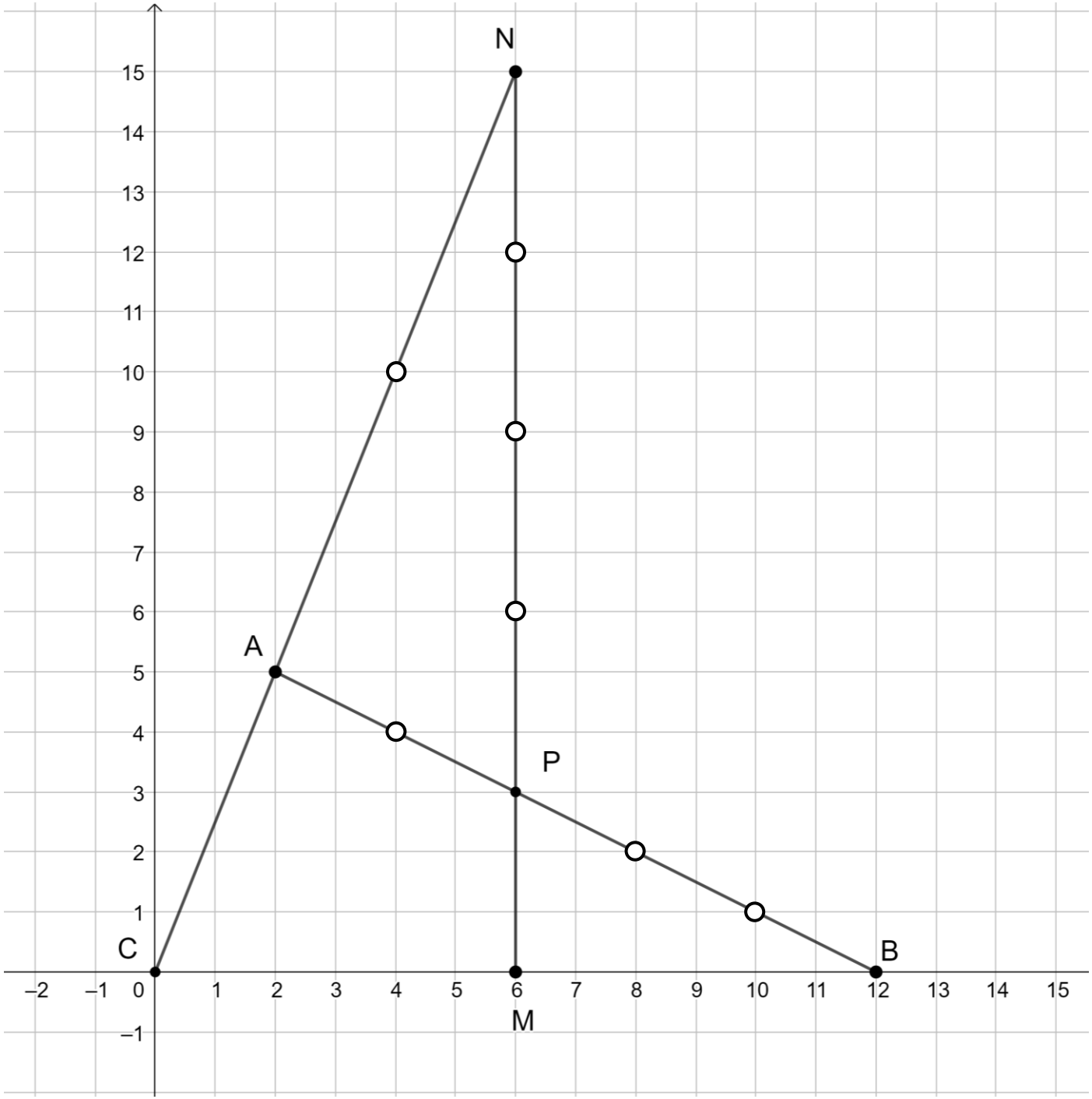
generalisation



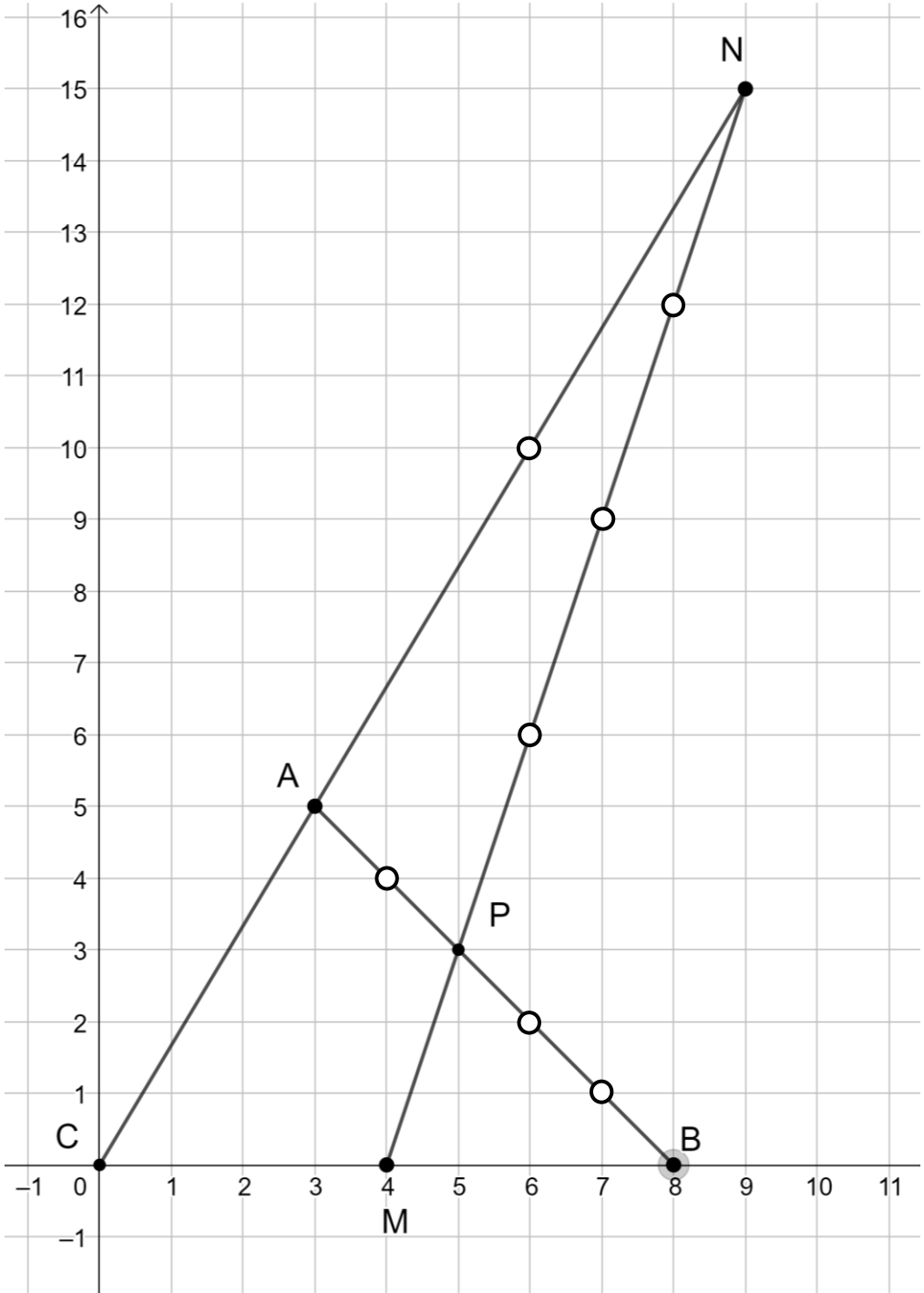
what if AT : TB = 1 : 2 ?

and CP : PT = g : h ?

further generalisation



what ratios are involved?



what ratios
are involved?

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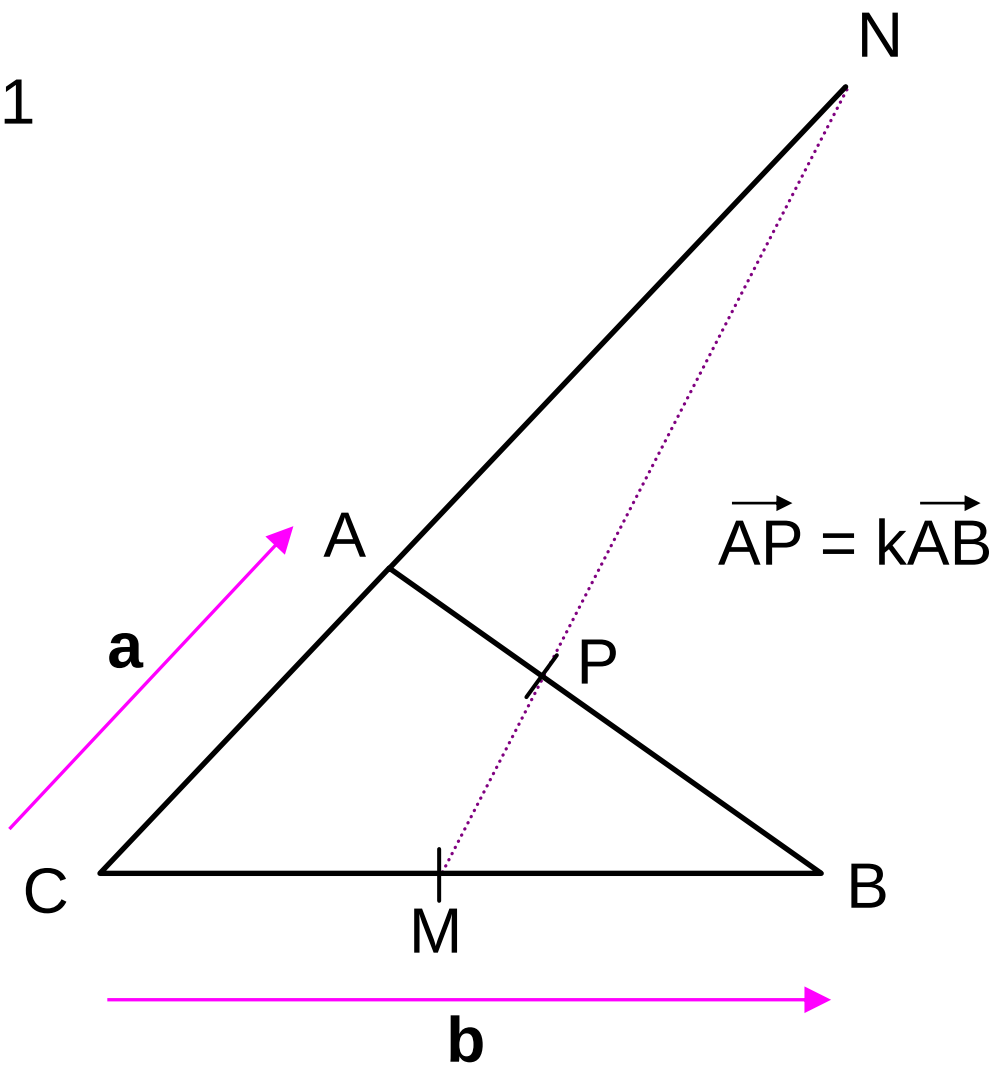


M = midpoint of CB

$\overrightarrow{CA}$ = **a**
$\overrightarrow{CB}$ = **b**

AN : CA = 2 : 1

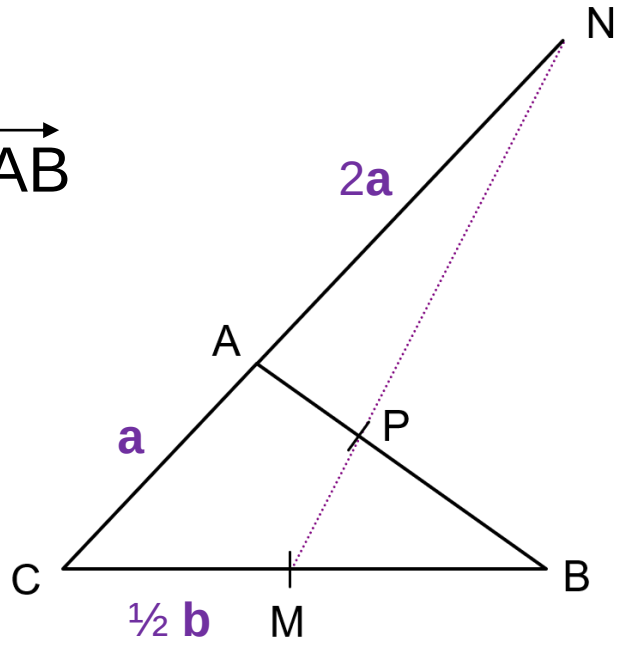
MPN is a straight line
work out the ratio AP : PB

$\overrightarrow{AN} = 2\mathbf{a}$ $\overrightarrow{CM} = \frac{1}{2}\mathbf{b}$ $\overrightarrow{AP} = k\overrightarrow{AB}$

$\overrightarrow{AB} = -\mathbf{a} + \mathbf{b}$ $\overrightarrow{AP} = -k\mathbf{a} + k\mathbf{b}$

$\overrightarrow{NP} = \overrightarrow{NA} + \overrightarrow{AP} = -2\mathbf{a} - k\mathbf{a} + k\mathbf{b}$

$\overrightarrow{NM} = \overrightarrow{NC} + \overrightarrow{CM} = -3\mathbf{a} + \frac{1}{2}\mathbf{b}$



$\overrightarrow{NM}$ is a multiple of $\overrightarrow{NP}$
since in the same direction

$$\frac{2+k}{3} = \frac{k}{\frac{1}{2}}$$

$$2 + k = 6k$$

$$2 = 5k \qquad k = 2/5$$

Edexcel mark scheme

$\overrightarrow{AB} = \mathbf{b} - \mathbf{a}$	1	This mark is given for finding a vector expression for $\overrightarrow{AB}$
$\overrightarrow{MN} = -\frac{1}{2}\mathbf{b} + \mathbf{a} + 2\mathbf{a}$ $= -\frac{1}{2}\mathbf{b} + 3\mathbf{a}$	1	This mark is given for finding a vector expression for $\overrightarrow{MN}$
$\overrightarrow{PN} = -k(\mathbf{b} - \mathbf{a}) + 2\mathbf{a}$ $= -k\mathbf{b} + (2 + k)\mathbf{a}$	1	This mark is given for finding a vector expression for $\overrightarrow{PN}$
Since $\overrightarrow{MN}$ is a multiple of $\overrightarrow{PN}$ $\frac{-\frac{1}{2}}{-k} = \frac{3}{(2+k)}$ $-\frac{1}{2}(2 + k) = -3k$	1	This mark is given for recognising that $\overrightarrow{MN}$ is a multiple of $\overrightarrow{PN}$ and comparing coefficients of **a** and **b**
$k = \frac{2}{5}$	1	This mark is given for the correct answer only

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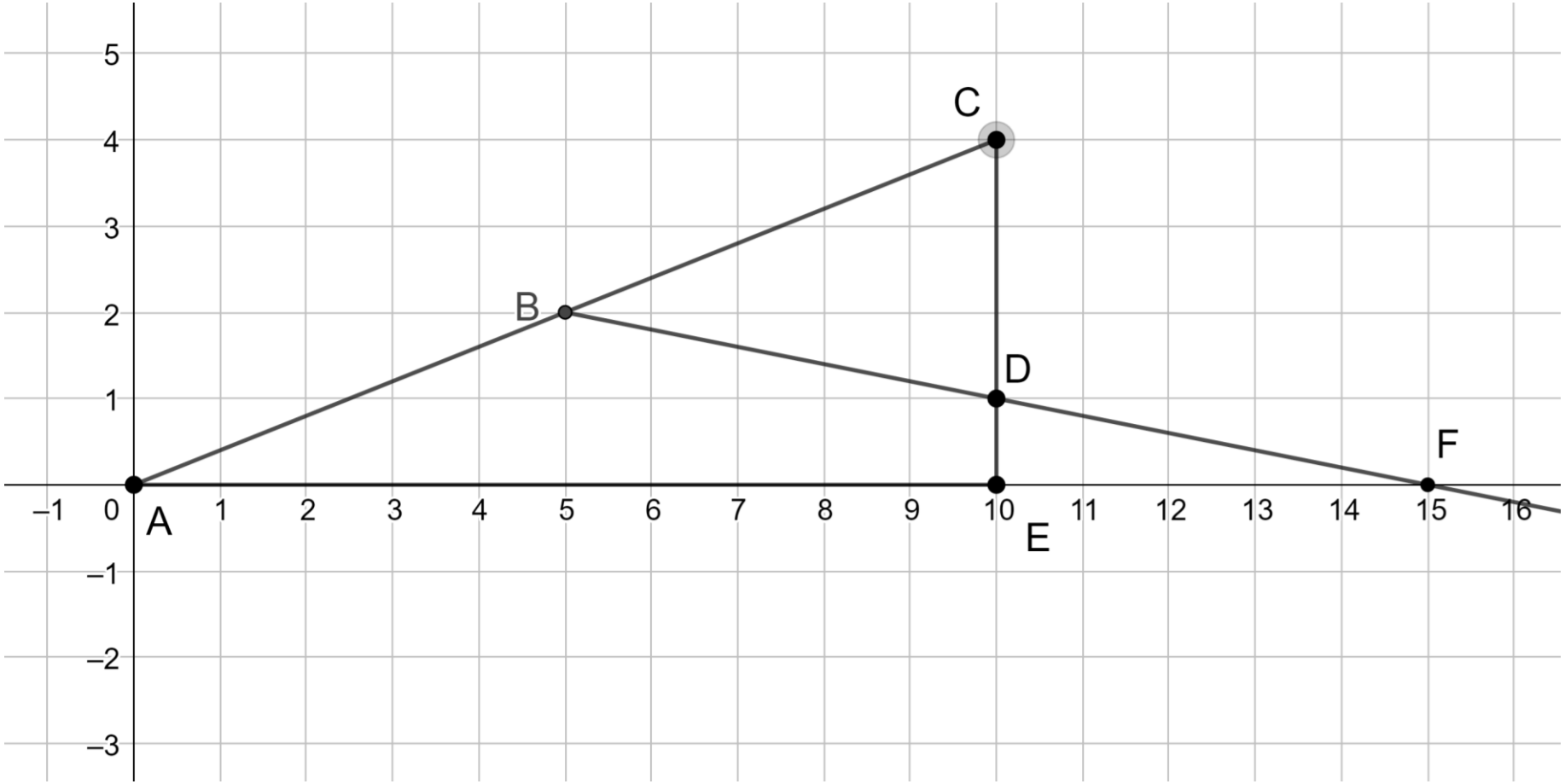
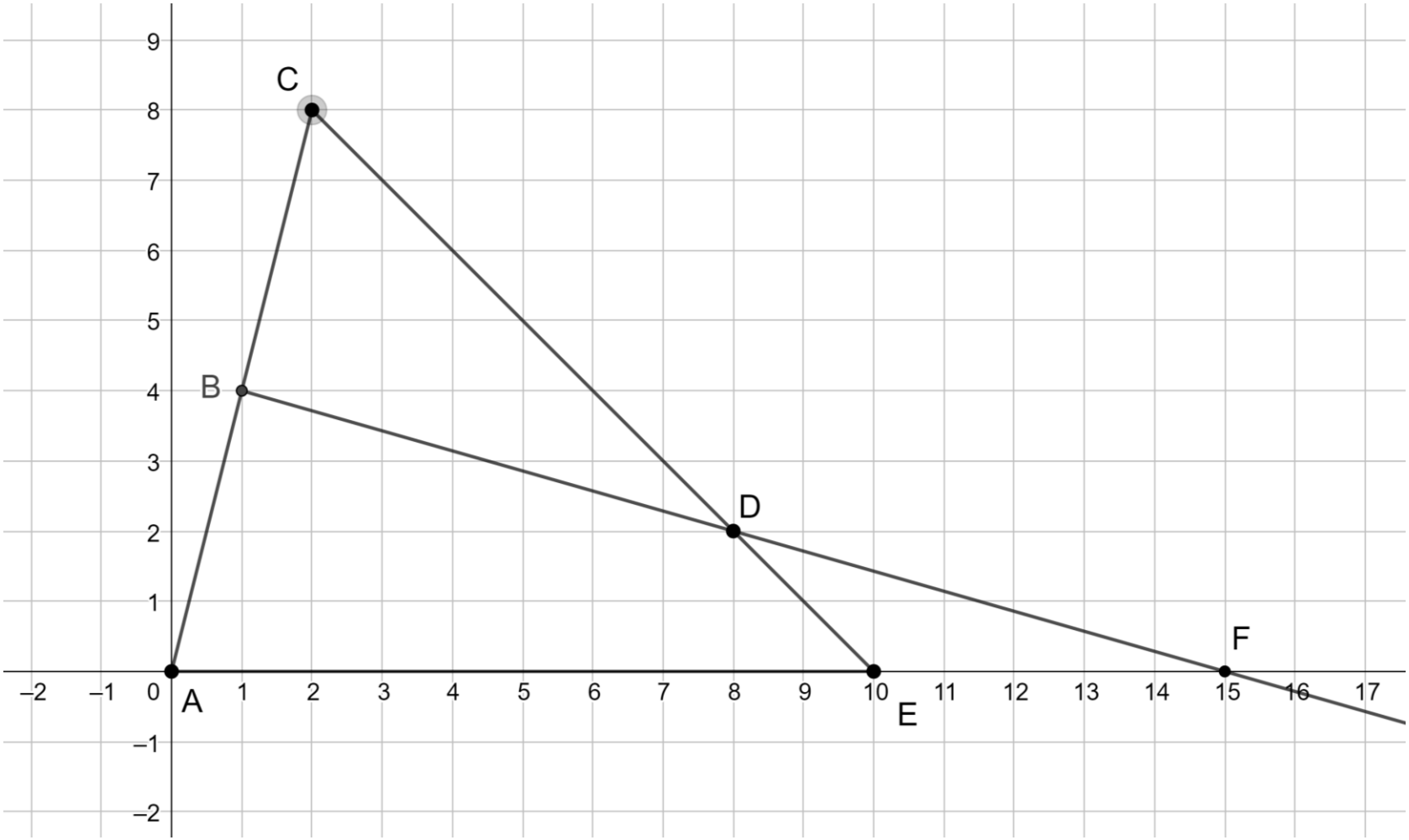
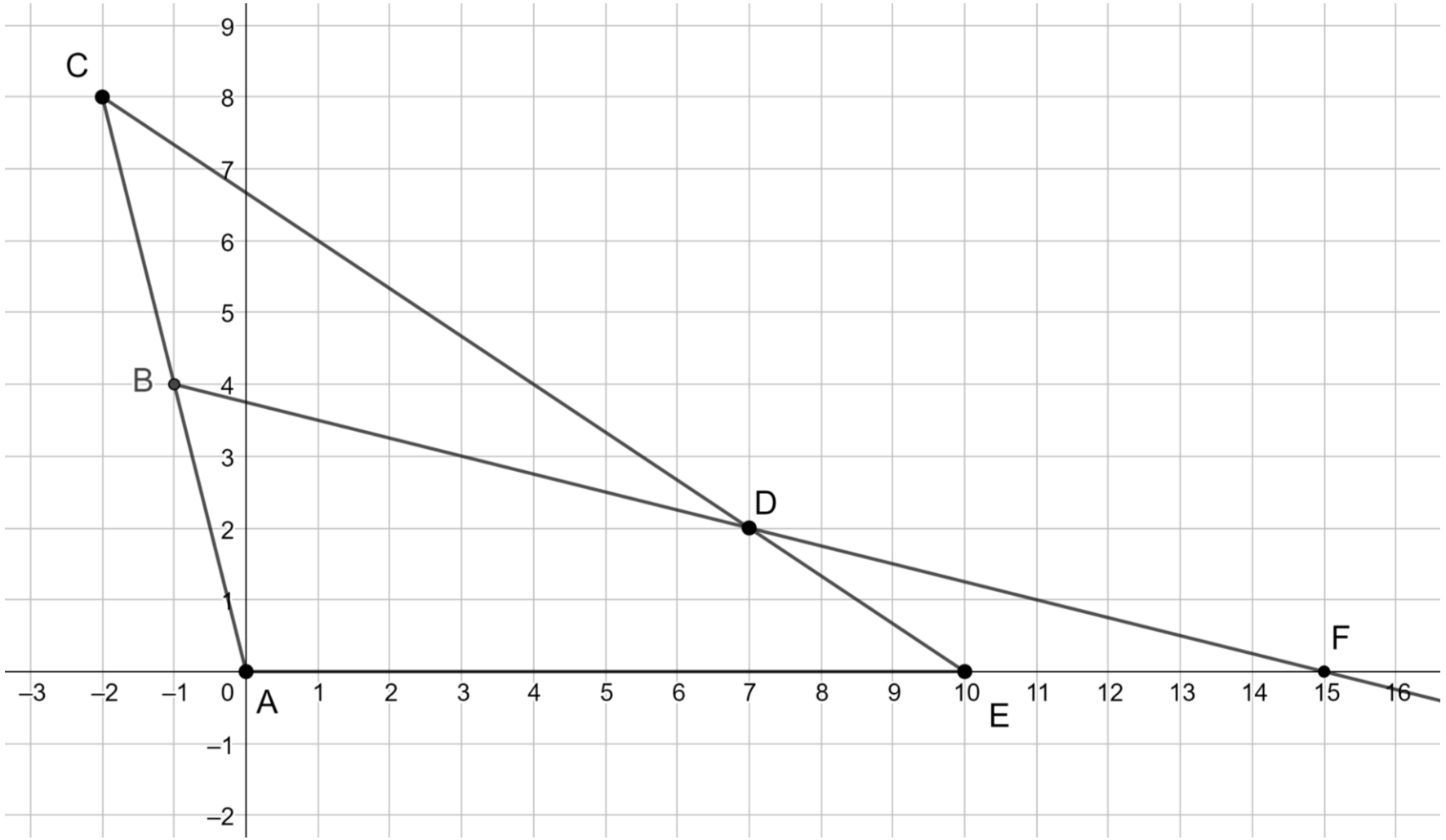


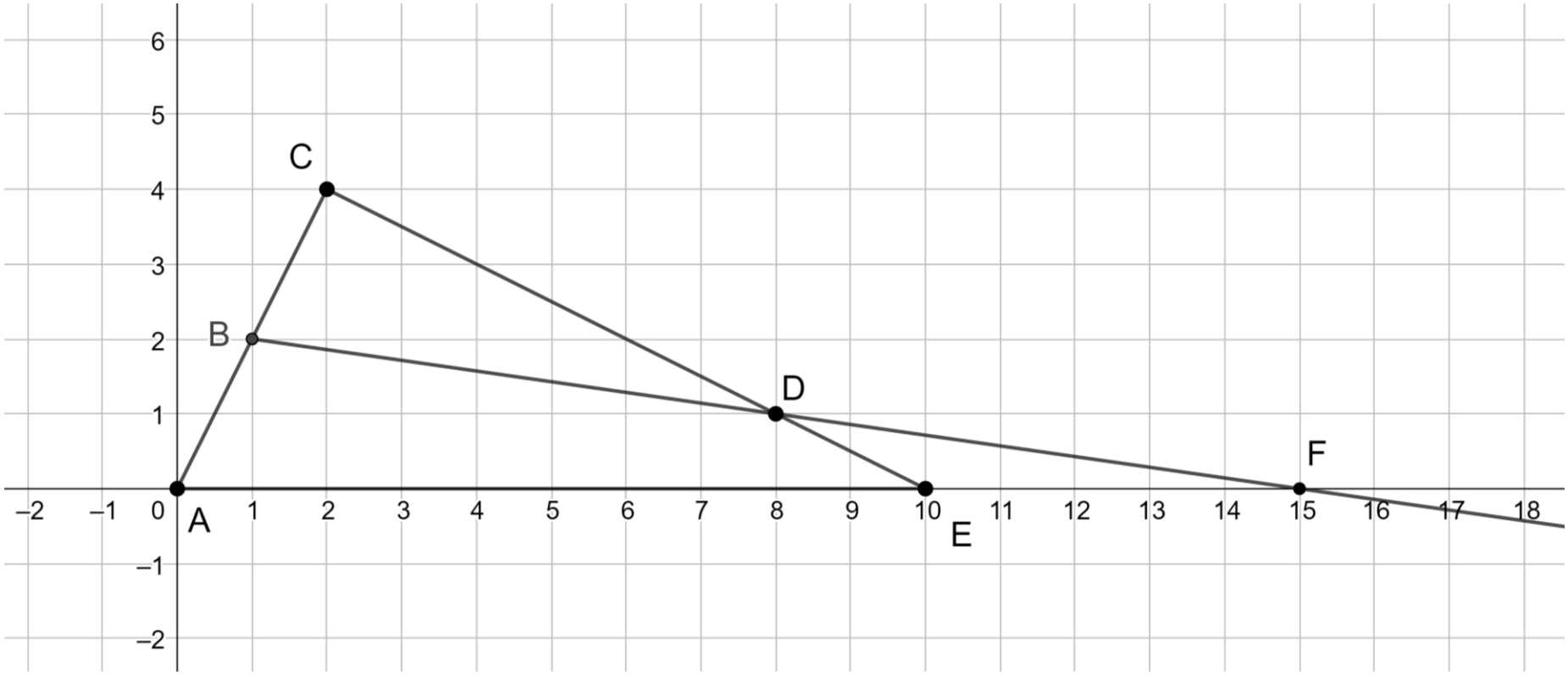
what ratios
are involved?



what ratios
are involved?

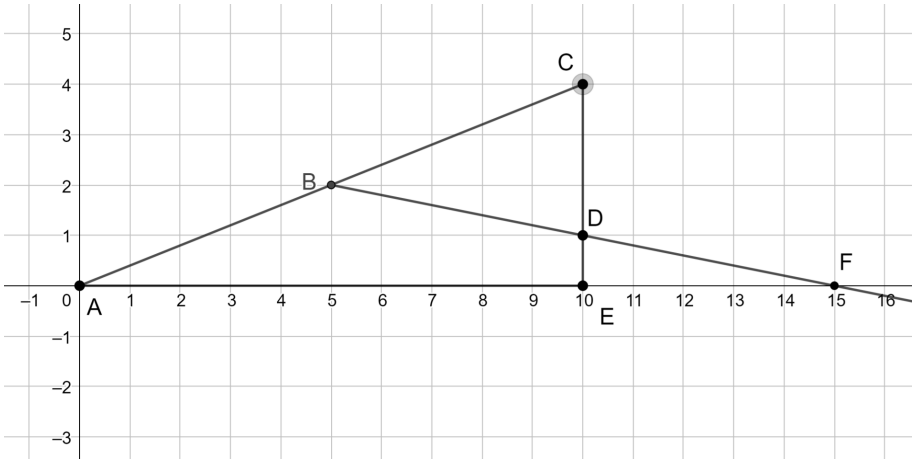


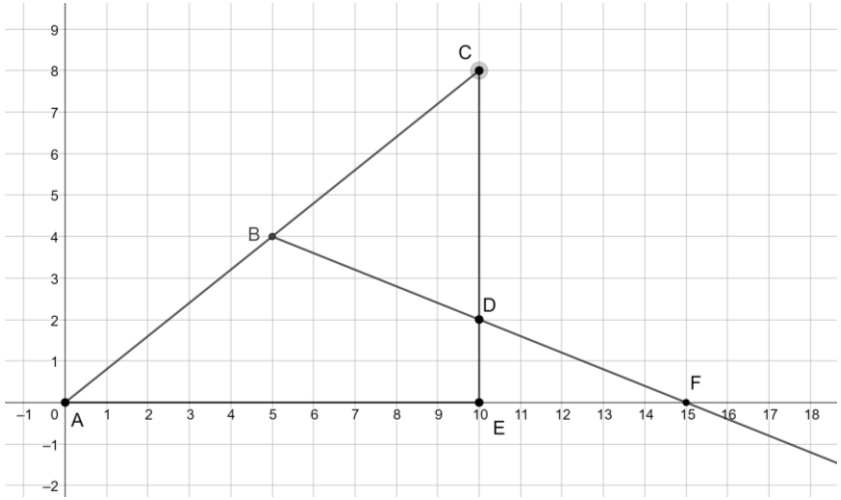
what ratios
are involved?



what ratios
are involved?

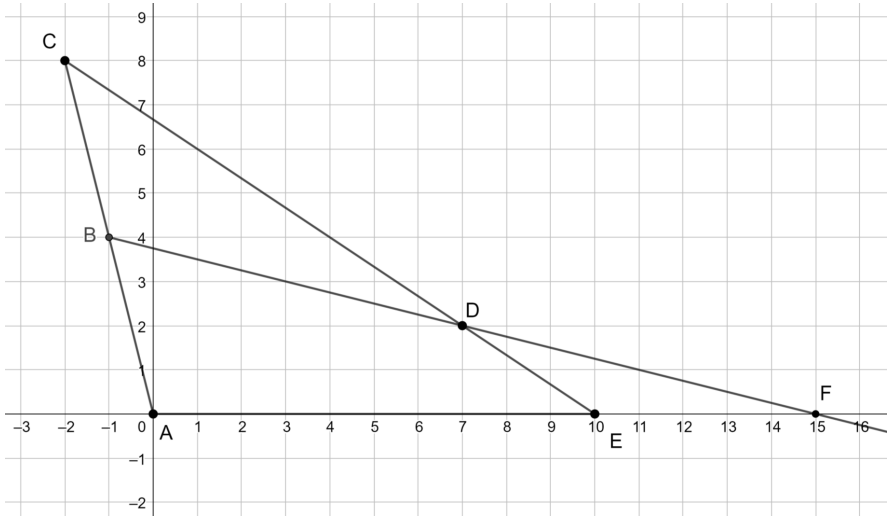
prove the general result

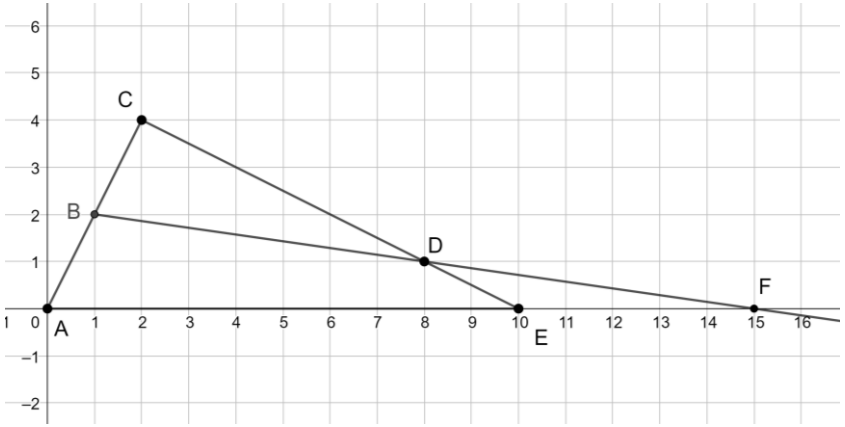




AB : BC = 1 : 1
ED : DC = 1 : 3
AE : EF = 2 : 1

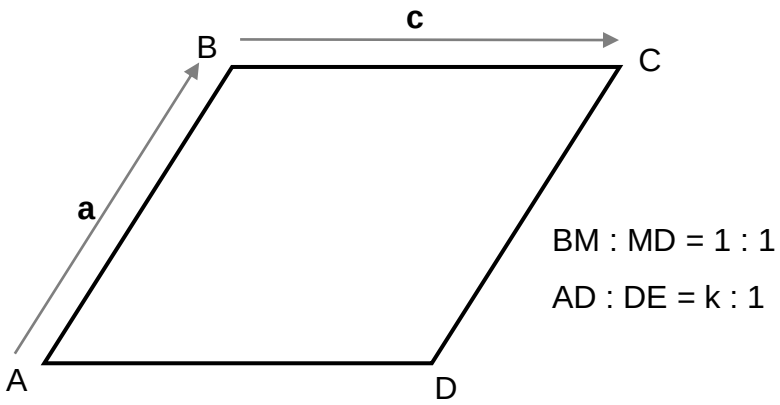
prove
BD : DF = 1 : 1





vector proofs

(1) a **parallelogram** (ABCD)

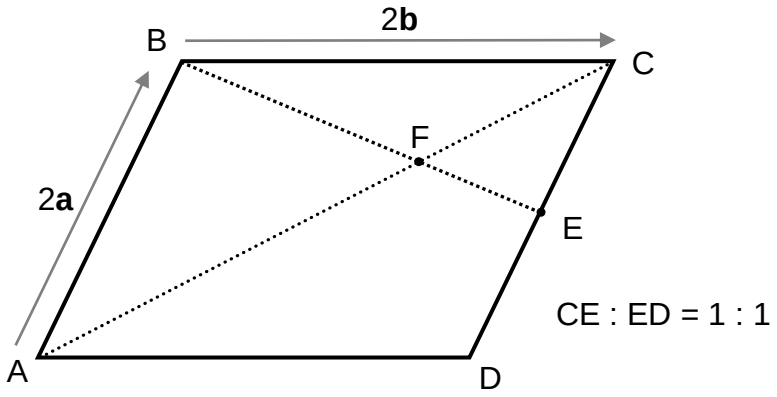


BM : MD = 1 : 1

AD : DE = k : 1

$\overrightarrow{ME} = 3\mathbf{c} - \frac{1}{2}\mathbf{a}$ find the value of k

(2) a **parallelogram** (ABCD)



CE : ED = 1 : 1

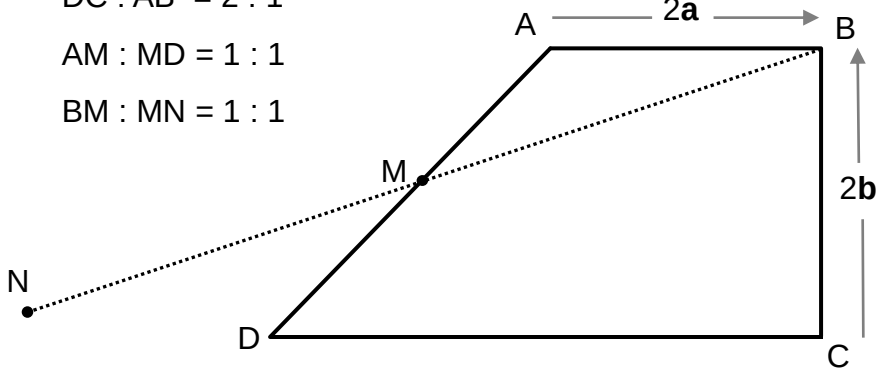
show that AF : FC = 2 : 1

(3) a **trapezium** (ABCD)

DC : AB = 2 : 1

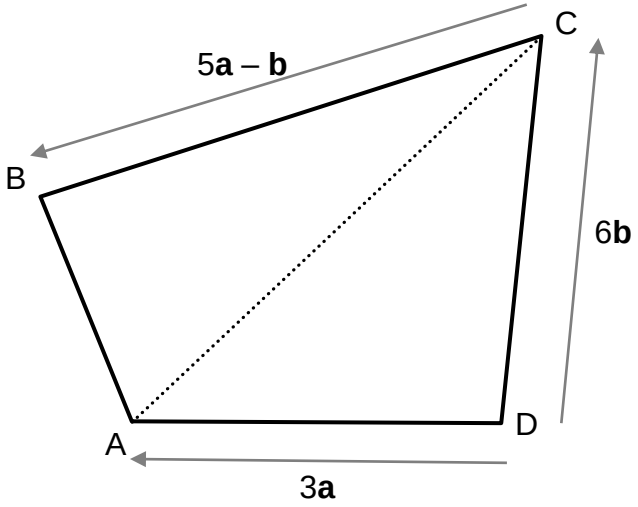
AM : MD = 1 : 1

BM : MN = 1 : 1



show that CDN is a straight line

(4)

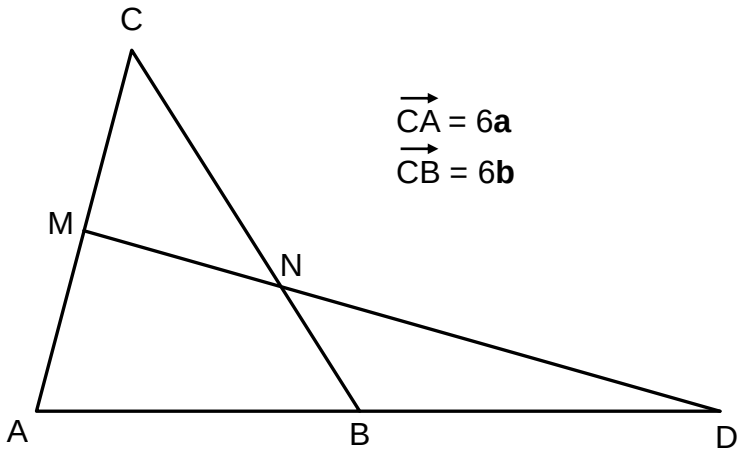


AX : XC = 1 : 2

DX : DB = k : 1 find the value of k

vector proofs

(1)

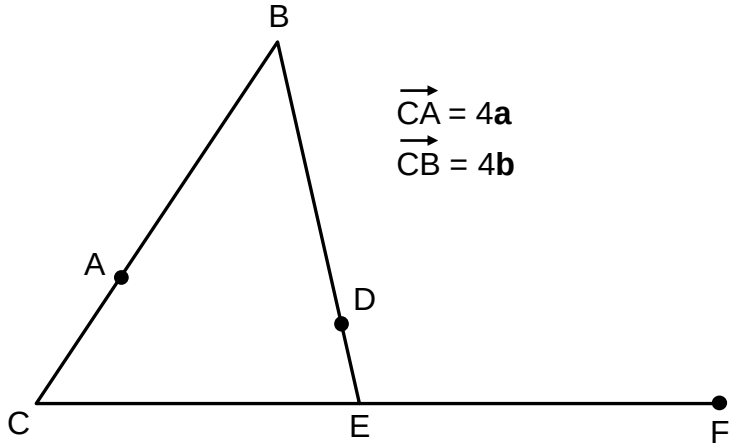


CM : MA = 1 : 1

AB : BD = 1 : 1

$\overrightarrow{CN} = k\mathbf{b}$ find the value of k

(2)



CA : AB = 1 : 2

ED : DB = 1 : 3

CE : EF = 1 : 2

show that ADF is a straight line

(3) choose some other ratios to explore