

## Sample Past Paper – Chapters 10 to 14 (AS level)

1. Find the following indefinite integral. Give your answer in its simplest form.

$$\int \left( \frac{3}{5}x^3 - 7\sqrt[3]{x^4 + 4} \right) dx =$$

2. Given that point C has position vector  $6i - 7j$  and point B has position vector  $-4i - 9j$

a) Find the vector  $\overrightarrow{AB}$

b) Find  $|\overrightarrow{AB}|$

c) Find the angle between the position vector of point A and the vector  $\overrightarrow{AB}$ .

3. Prove the following equations:

a)  $e^{2\ln(x)} + 10^{\log(7)} + \ln e = x^2 + 8, x > 0$

b)  $\ln(e^{2x+1}) + 10^{\log(x)} + \log 1 > 0, x > 0$

4. Point  $M(a, b)$  lies on the straight line with equation  $y = 3x + 1$ . Given that  $a = \log(k)$  and  $b = \log(m)$ , show that  $m = 10k^3$  where  $m > 0$  and  $k > 0$ .

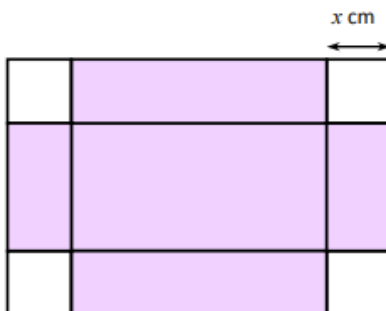
5. Prove from first principles that the derivative of  $\frac{1}{x}$  is  $-\frac{1}{x^2}$

6. Show that given an original rectangle of card measured  $80\text{cm}$  by  $50\text{cm}$ , if the squares are removed from each corner (with sides  $x\text{ cm}$  long), then the volume of the box is given by:

$$V = 4x^3 - 260x^2 + 4000x$$

a) Find the value of  $x$  that will give the maximum possible volume.

b) Calculate the maximum possible volume.



7. a) Solve the equation  $\sin^2 x + 3 \sin x \cos x + 2 \cos^2 x = 0$  in the range  $0 \leq x < 360^\circ$

b) i) What is the value of  $f(x) = (4 \sin^2 x + 4 \cos x + 1)^2$ .

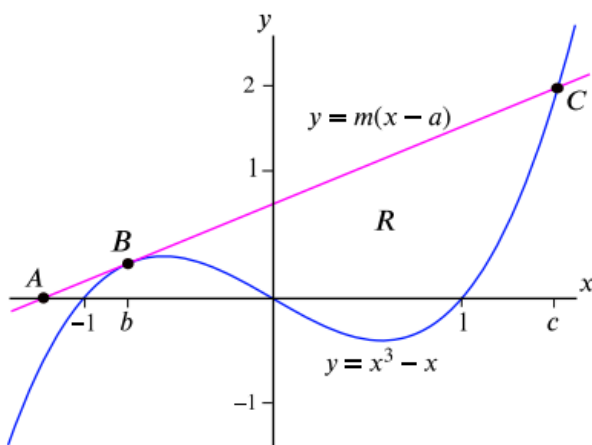
ii) At which value of  $x$  does it occur?

8. The gradient of the curve  $y = 3x^2 + 5x - 12$  is 17 at the point  $P$ . Calculate the coordinates of  $P$ .

The curve cuts the  $x$ -axis at  $Q$  and  $R$ . Find the gradient of the curve at  $Q$  and at  $R$ .

9. The graphs of  $y = x^3 - x$  and  $y = m(x - a)$  are drawn on the axes below. Here  $m > 0$  and  $a \leq -1$ .

The line  $y = m(x - a)$  meets the  $x$ -axis at  $A = (a, 0)$ , touches the cubic  $y = x^3 - x$  at  $B$  and intersects again with the cubic at  $C$ . The  $x$ -coordinates of  $B$  and  $C$  are respectively  $b$  and  $c$ .



(i) Use the fact that the line and cubic *touch* when  $x = b$ , to show that  $m = 3b^2 - 1$ .

(ii) Show further that

$$a = \frac{2b^3}{3b^2 - 1}.$$

(iii) If  $a = -10^6$ , what is the approximate value of  $b$ ?