Functions (Inverse and Composite)

1. Given that $f: x \to 4 - x^2$
   (a) State the maximum value of f.
   (b) Write down the range of f.

2. Draw the curves and hence find the range and domain of the following functions:
   (a) $f(x) = x^2 - 2x - 8$
   (b) $g(x) - 2x^2 - 8x + 3$
   (c) $h(x) = -x^2 + 4x - 4$

3. Find the inverse function of the following:
   (a) $f: x \to 3x$
   (b) $f: x \to 3x - 4$
   (c) $f: x \to \frac{x-3}{2}$

4. Given that $f: x \to \frac{4}{2x-1}$ and $g: x \to \frac{x}{x+4}$,

   (a) State the values of x which must be excluded from the domain of f and g.

   (b) Find the value of (i) $g(2)$ (ii) $fg(2)$

   (c) Find f^{-1} and g^{-1}.

5. The functions of f and g are defined by

$$f: x \to 2x - 1 \quad \text{and } g: x \to x^2 + 3$$

   (a) Find the domain and range of g.
   (b) Find the values of x for which $f(10) = g(x)$
   (c) Copy and complete the following: (i) $f^{-1}: x \to$....
   (ii) $gf: x \to$....
   (iii) $fg: x \to$....

6. The functions *f*, *g*, and *h* are defined by:

$$f(x) = x + 3,\ \ g(x) = \tfrac{1}{2}x,\ \ h(x) = x^2.$$

(a) Find:

(i) $gf(x)$
(ii) $fg(x)$
(iii) $f^{-1}(x)$
(iv) $hf(x)$

(b) Solve the equations:

(i) $f(x) = -8$
(ii) $hf(x) = 16$
(iii) $hf(x) + g^{-1}(x) = 0$

Give your answers to 2 decimal places where necessary.

7. Give the range of the function $f: x \to \frac{2}{x-3}$ over the domain {4, 5, 6, 7}.

8. Given that $f(x) = 2x - 1$ and $g(x) = 3 - 2x$,

(a) Solve the equation $f(x) = x$.

(b) Find the function $(fg)^{-1}$ and hence prove that $(fg)^{-1} = g^{-1} \cdot f^{-1}$.

(c) Find the value of the following: **(i)** $gf(1)$
(ii) $f^{-1}(3)$
(iii) $ff(0)$
(iv) $fg^{-1}(-1)$

9. Two functions *f* and *g* are defined by:

$$f: x \to \frac{3}{x} \text{ and } g: x \to 4 - x$$

(a) Given that $h = fg$, write down the function h in the form $h: x \to$....

(b) Hence find $h^{-1}: x \to$....

(c) Find the two values of *x* for which *gf* = *h*.

(d) Given that $f(a) = b$ show that $f(b) = a$.